2 Binary Image Analysis

Binary image

- Each pixel takes one of two values: 0 or 1.
- Usually, 0 denote background, 1 denote foreground (or object).
- Can be obtained from gray image by thresholding.

2.1 Thresholding

\[ b(r, c) = \begin{cases} 
1 & \text{if } g(r, c) > \Gamma \\
0 & \text{otherwise.} 
\end{cases} \]  

(1)

- \( g(r, c) \) is the gray value of pixel at \((r, c)\).
- \( b(r, c) \) is the binary value of pixel at \((r, c)\).
- \( \Gamma \) is the threshold.
- Pixels with gray values above threshold are set to 1; others are set to 0.
How to know what threshold to use?

Look at the histogram $h$ of gray image $g(r, c)$.

$$h(m) = |\{(r, c) \mid g(r, c) = m\}|$$

- $h(m)$ = number of pixels with gray value $g(r, c)$ equal to $m$.
- Pick a threshold between major peaks in the histogram.

**Number of pixels**

![Image](image.png)
Example 2.1.2: Effects of thresholding.

(a) Original  
(b) Threshold at 100

(c) Threshold at 128  
(d) Threshold at 150

Further reading:
- Shapiro, Section 3.8.2 Automatic Thresholding

What did you observe?
Where should you set the threshold?
2.2 Morphological Operations

What are they?

- Morphology means form and structure.
- Morphological operations change the form or shape of objects in images.
- Morphological operations are defined on two elements:
  - a binary image $B$
  - a binary structuring element $S$

Examples of structuring elements ($\bullet = 1$):

(a) Box(3,5)  (b) Disk(5)  (c) Ring(5)

Morphological operations are expressed mathematically in terms of sets that contain the locations of pixels with value 1:

$$X = \{x \mid b(x) = 1\}$$  \hspace{1cm} (3)

- $X$ is an image or a structuring element.
- $x = (r, c)$ is the pixel location at row $r$, column $c$.
- $b(x)$ is the binary value of the pixel at $x$.

They are four basic operations:

- dilation
- erosion
- closing
- opening

which are defined in terms of translation.

Translation

Translation $X_t$ of a set of pixels $X$ by a position vector $t$ is

$$X_t = \{x + t \mid x \in X\}$$  \hspace{1cm} (4)

Example 2.2.1: Translation.
Dilation

Dilation $B \oplus S$ of binary image $B$ by structuring element $S$ is

$$B \oplus S = \bigcup_{b \in B} S_b$$

(5)

- $S$ has an *origin*, usually the center pixel.
- Origin of $S$ is initially located at $(0, 0)$ of $B$.
- $S_b$ is a translation of $S$ by $b \in B$.
- $B \oplus S$ is the union of all $S_b$.
- Same as placing $S$ at every 1-pixel of $B$ and taking the union.
- The result is the union.

Example 2.2.2: Dilation.

(a) $S$ at $(0, 0)$ of $B$

(b) $S_{(2,2)}$

(c) $S_{(3,2)}$

(d) $B \oplus S = S_{(2,2)} \cup S_{(3,2)}$
Erosion

Erosion $B \ominus S$ of binary image $B$ by structuring element $S$ is

$$B \ominus S = \{ b \mid b + s \in B \forall s \in S \}$$

- Place $S$ at any location $b$ in $B$.
- If every 1-pixel of $S$ covers a 1-pixel of $B$,
  then put a 1-pixel in location $b$ of the result.

Example 2.2.3: Erosion.

- (a) $S$ at $(0, 0)$ of $B$
- (b) $S_{(2,2)}$
- (c) $S_{(3,2)}$
- (d) $B \ominus S = \{(2, 2), (3, 2)\}$

Examples 2.2.2 and 2.2.3 suggest that $(B \ominus S) \ominus S = B$.
Is it true in general? No.

Closing

Closing $B \bullet S$ of binary image $B$ by structuring element $S$ is

$$B \bullet S = (B \ominus S) \ominus S$$

Example 2.2.4: Closing.
Opening

Opening $B \circ S$ of binary image $B$ by structuring element $S$ is

$$B \circ S = (B \ominus S) \oplus S$$  \hspace{1cm} (8)

Example 2.2.5: Opening.

Exercise: What have you observed from the examples?

Dilation

Erosion
Closing

Opening

Application Example 2.2.6: Extracting regions in medical image.

(a) Original image
(b) Thresholding at 128
(c) Closing with Disk(5)
(d) Opening with Disk(11)
How to do all the magic easily?
Use proper tools. I used Cantata for the examples.
You will be using a newer version of Cantata for lab exercises.

Further reading:
- Shapiro, Section 3.5.3 Some Applications of Binary
  Morphology.