Question (1)

(i) Using TDM, channel speed = 5 Kb/s or \( \mu = 5 \) packets/sec
   Arrivals = 150 packets/min or \( \lambda = 2.5 \) packets/sec
   \( \rho = \lambda / \mu = 0.5 \)
   Consider a single channel,
   Average number of packets in system = \( N = \rho / (1 - \rho) = 0.5/0.5 = 1 \)
   Average number of packets in queue = \( N_q = \rho \times N = 0.5 \)
   Average delay per packet = \( 1/(\mu - \lambda) = 1/(5 - 2.5) = 0.4 \) sec

(ii) Using Statistical Multiplexing, channel speed = 50 Kb/s or \( \mu = 50 \) packets/sec
   Arrivals = 1500 packets/min or \( \lambda = 25 \) packets/sec
   \( \rho = \lambda / \mu = 0.5 \)
   Average number of packets in system = \( N = \rho / (1 - \rho) = 0.5/0.5 = 1 \)
   Average number of packets in queue = \( N_q = \rho \times N = 0.5 \)
   Average delay per packet = \( 1/(\mu - \lambda) = 1/(50 - 25) = 0.04 \) sec

Question (2)

For M/M/1/N queue, \( P_B = (1-\rho)\rho^N / (1 - \rho^{N+1}) \)
For large enough \( N \) and \( r \), \( P_B \) can be approximated as \( (1-\rho)\rho^N \)
\( N = \log \left( P_B / (1-\rho) \right) / \log \rho \)
\( P_B = 10^{-3}, \rho = 0.8, N = 23.7 \) or 24
\( P_B = 10^{-6}, \rho = 0.8, N = 54.7 \) or 55
\( P_B = 10^{-9}, \rho = 0.8, N = 85.7 \) or 86

Question (3)

(i) For waiting customers, \( \lambda = 6/\text{min}, T = 5\text{min}, \)
    Using Little’s Theorem, \( N = \lambda \times T = 30 \)

(ii) For eating customers, \( \lambda = 0.5 \times 6 = 3, T = 30\text{min}, N = 3 \times 30 = 90 \)

Question (4)

Minimum transmission time occurs when packets from one stream arrives immediately
after the departure (service completion of a packet from another stream) because there
will be no waiting. Average system time is the transmission time which is \( T/2/ \)

Maximum transmission time occurs when packets from the two streams arrived at the
same time. Assume packets from stream 1 are always served first. Average system time is
\( 0.5 \times (\text{service time of packet 1 and 2} + \text{waiting time of packet 2}) = (T + T/2)/2 = 3T/4. \)
\[ \text{Var}(W) = E((W - E(W))^2) \]

Again, minimum transmission time occurs when packets from one stream arrives immediately after the departure. There is no waiting and variance of waiting time is 0.

Again, maximum waiting time variation occurs for packet from stream 2 when packets from the two streams arrived at the same time. \[ \text{Var}(W) = 0.5 (T - 3T/4)^2 + 0.5 (T/2 - 3T/4) = (T^2/16). \]

**Question (5)**

Load = 30 * 3 = 90 Erlang, blocking = 2%
Using the Erlang B formula or table, # of circuits = 103.

**Question (6)**

It happens that M/G/m/m systems have the same service time probability distribution as a M/M/m/m systems (see Bertsekas and Gallager: Section 3.4.3). Hence, using the Erlang B formula for M/M/m/m, the blocking will be about 9.6%.

**Question (7)**

On the average, in 10ms, the slowest rate that the buffer can be drained, which is 5Mbps. Hence, in 10ms, only up to 50Kb can be drained.

**Question (8)**


You should have to write down the equations for the PDF used, Exponential, \(0.5 e^{-(x/2)}\) and Pareto, \(2x^{-3}\)
We can use the global balance equation, which states that at equilibrium, to write down the relationships for the states \{0,1′,2′,…, (k-1)′\}. The probability of transition out of a state equals to the probability of a transition into the same state. The global balance equations are:

\[ \lambda P_i = \lambda P_{i+1}, \quad i = 0,1,2,…,k-2 \]

Therefore, \[ P_0 = P'_1 = P'_2 = … = P'_{(k-1)} \]

Also, \[ P_1 = \frac{\lambda}{\mu} P_0 = \rho P_0 \]

where \[ \rho = \frac{\lambda}{\mu} \]

Using global balance equations, which state that frequency of transition out of a (set of) state(s) is the same as transition into the same (set of) state(s).

\[ \lambda (P_1 + P'_1) = \mu P_2, \quad …, \quad \lambda (P_{k-1} + P'_{k-1}) = \mu P_k \]

Therefore, \[ P_2 = \rho P_1 + P_0, \quad P_3 = \rho^2 P_1 + (1+\rho) P_0, \quad …, \quad P_k = \rho P_{k-1} + P_0 \]

or \[ P_i = (\rho^i + (1-\rho^{-1})/(1-\rho)) P_0, \quad i = 1,2,3,4,…k \]

Finally, we can see that the detail balance equations apply for the rest of the states with \( k \) or more customers (there is no loop). Therefore, \( P_j = \rho^{(j-k)} P_{j-1}, \quad j=k+1,k+2,…, \)

Now \[ kP_0 + \sum_{i=1}^{k} P_i + \sum_{j>k} P_j = 1 \] (note that all terms can be expressed in terms of \( P_0 \))

If you simplify the expression, you will find that \[ P_0 = (1-\rho)/K \] and \[ N = (k-1)/2 + \rho/(1-\rho) \]

Finally, \[ T = N/\lambda \]

**Question (10)**

No “correct” answers. You can refer to the lecture notes/reference books for possibilities/ideas.