Appendix

Proof of Observation 1. Differentiating (4) with respect to $c$,

$$-[v + h] \chi \frac{d^2 \alpha}{df^2} \frac{\partial f}{\partial c} = n,$$

and hence, by (1),

$$\frac{\partial f}{\partial c} = \frac{n}{-[v + h] \chi \frac{d^2 \alpha}{df^2}} < 0. \quad (A1)$$

Differentiating (4) with respect to $n$,

$$-[v + h] \chi \frac{d^2 \alpha}{df^2} \frac{\partial f}{\partial n} = c,$$

and hence, by (1),

$$\frac{\partial f}{\partial n} = -\frac{c}{-[v + h] \chi \frac{d^2 \alpha}{df^2}} < 0. \quad (A2)$$

Differentiating (4) with respect to $\chi$,

$$-[v + h] \frac{d\alpha}{df} - [v + h] \chi \frac{d^2 \alpha}{df^2} \frac{\partial f}{\partial \chi} = 0,$$

and hence, by (1),

$$\frac{\partial f}{\partial \chi} = \frac{\frac{d\alpha}{df}}{\chi \frac{d^2 \alpha}{df^2}} > 0. \quad (A3)$$

By (2), $d\chi / dK > 0$, hence $\partial f / \partial K > 0$. If $K = 0$, then $\chi(K) = 0$, hence by (4),

$f(n|K = 0) = 0$, $\forall n \in [0,1]$. Further, if $K \to \infty$, $\chi(K) \to 1$, hence, by (4),

$$\lim_{K \to \infty} f(n) = f_\infty(n). [ ]$$
Proof of Observation 2. We first prove that \( B(n) \) is monotone decreasing in \( n \). Consider \( n_1 \) and \( n_2 \) such that \( n_1 < n_2 \). Let user \( n_1 \) choose the precautions, \( f(n_2) \), associated with user \( n_2 \). Since \( n_1 < n_2 \), her expected net benefit would be

\[
v - [v + h] \alpha(f(n_2)) \chi - p - n_1 cf(n_2)
\]

By (3), the precautions \( f(n_i) \) must provide user \( n_1 \) with the maximum expected net benefit, and, in particular,

\[
B(n_1 | K) = v - [v + h] \alpha(f(n_1)) \chi - p - n_1 cf(n_1) \\
\geq v - [v + h] \alpha(f(n_2)) \chi - p - n_2 cf(n_2).
\]

Hence, by (A4) and (A5), \( B(n_1 | K) > B(n_2 | K) \), which is the result.

Since \( B(n) \) is monotone decreasing in \( n \), the demand for the service is characterized as follows. Consider the most sophisticated user, \( n = 0 \). By (3), her cost of precaution is zero and therefore she will choose the highest precaution, i.e., \( f(0) \rightarrow \infty \). Under the assumption that \( v > p \) and by (1), the most sophisticated user would buy since \( B(0) = v - p > 0 \).

Consider the most naïve user, \( n = 1 \). If \( B(1) \geq 0 \), then, \( B(n) > 0 \) for all \( n < 1 \) and all other users would buy. However, if \( B(1) < 0 \), the most naïve user does not buy the service, and there exists some critical level as claimed. [ ]

Proof of Observation 3. Differentiating (5) with respect to \( c \),

\[
-[v + h] \chi \frac{d \alpha(f(\hat{n}))}{df} \left[ \frac{\partial f(\hat{n})}{\partial c} + \frac{\partial f(\hat{n})}{\partial n} \frac{\hat{n}}{dc} \right] - \hat{n} f(\hat{n}) - cf(\hat{n}) \frac{\hat{n}}{dc} - c\hat{n} \left[ \frac{\partial f(\hat{n})}{\partial c} + \frac{\partial f(\hat{n})}{\partial n} \frac{\hat{n}}{dc} \right] = 0
\]

hence, using (4),

\[
\frac{\partial \hat{n}}{\partial c} = -\frac{\hat{n}}{c} < 0,
\]

i.e., the marginal user, \( \hat{n} \), is decreasing in \( c \).
Differentiating (5) with respect to \( p \),

\[
-[v + h] \frac{d \alpha}{df} \left[ \frac{\partial f}{\partial p} + \frac{\partial f}{\partial n} \frac{\partial \hat{n}}{\partial p} \right] - c f (\hat{n}) \frac{\partial \hat{n}}{\partial p} - n c f (\hat{n}) \frac{\partial^2 \hat{n}}{\partial n \partial p} = 0,
\]

hence, using (4),

\[
\frac{\partial \hat{n}}{\partial p} = -\frac{1}{c f (\hat{n})} < 0 \tag{A7}
\]

Differentiating (5) with respect to \( K \),

\[
-[v + h] \frac{d \alpha}{df} \left[ \frac{\partial f}{\partial K} + \frac{\partial f}{\partial n} \frac{\partial \hat{n}}{\partial K} \right] - [v + h] \alpha (f (\hat{n})) \frac{d \chi}{dK} - c f (\hat{n}) \frac{\partial \hat{n}}{\partial K} - n c f (\hat{n}) \frac{\partial^2 \hat{n}}{\partial n \partial K} = 0,
\]

hence, using (4),

\[
\frac{\partial \hat{n}}{\partial K} = -\frac{[v + h] \alpha (f (\hat{n})) \frac{d \chi (K)}{dK}}{c f (\hat{n})} < 0 \tag{A8}
\]

Further differentiating (A8) with respect to \( K \),

\[
\frac{\partial^2 \hat{n}}{\partial K^2} = -\frac{[v + h] \alpha (f (\hat{n})) \frac{d^2 \chi}{dK^2}}{c f (\hat{n})} - \frac{[v + h] \alpha (f (\hat{n})) \frac{d \chi}{dK}}{c f^2 (\hat{n})} \frac{\partial f}{\partial K} - c f (\hat{n}) \frac{\partial^2 \hat{n}}{\partial n \partial K} \left[ f \frac{d \alpha}{df} \right] - \alpha > 0, \tag{A9}
\]

which follows from (1), (2), (A2), (A3), and (A8). By (A8) and (A9), \( \hat{n} \) is decreasing and convex in \( K \).

If \( K \to 0 \), then \( \chi (K) \to 0 \). Hence by (3), users’ expected net benefit,

\[
B(n) \to v - p - n c f (n),
\]

which is maximized with \( f (n) = 0 \). Thus \( B(n) \to v - p \), for all \( n \).

Since \( v > p \), all users buy the service. Accordingly, if \( K \to 0 \), then \( \hat{n} \to 1 \).

Now, if \( K \to \infty \), then \( \chi (K) \to 1 \), hence, by (3), users’ expected net benefit,

\[
B(n) \to v - [v + h] \alpha (f (n)) - p - n c f (n).
\]

As proved by Observation 2, the most sophisticated user would buy the service, i.e., \( B(0 | K \to \infty) > 0 \). Consider the user with \( n = 1 \). If her
expected net benefit, \( B(l) \rightarrow v-[v+h] \alpha(f(l))-p-cf(l) \geq 0 \), then by Observation 2, 
\( B(n) > 0 \) for all \( n \). Hence all users will buy the service. Otherwise, if \( B(l) < 0 \), then there exists some \( \hat{n}_0 \) such that 
\[
B(\hat{n}_0) \rightarrow v-[v+h] \alpha(f(\hat{n}_0))-p-\hat{n}_0 cf(\hat{n}_0) = 0 ,
\]
which completes the proof.

**Proof of Observation 4.** To simplify notation, define 
\[
A(K) = \int_{\Phi}^{\hat{n}(K)} \alpha(f(n)|K)d\Phi(n) .
\]
Since \( d\alpha/df < 0 \), then \( \hat{\partial A}/\hat{\partial f} < 0 \). Further \( \hat{\partial A}/\hat{\partial n} > 0 \). Substituting (A10) in (8), and then differentiating with respect to \( \eta \),
\[
e A \left[ - \frac{d\chi}{dk_i} + [1-\eta] \frac{d^2\chi}{dk_i^2} \alpha \right] = \frac{d^2c_k}{dk_i^2} \frac{\partial k_i}{\partial \eta} ,
\]
which simplifies to
\[
\frac{\partial k_i}{\partial \eta} = \frac{e \frac{d\chi}{dk_i} A}{e[1-\eta]A \frac{d^2\chi}{dk_i^2} - \frac{d^2c_k}{dk_i^2}} < 0 . \tag{A11}
\]
Similarly, differentiating (8) with respect to \( \hat{n} \),
\[
e [1-\eta] \left[ \frac{\partial A}{\partial \hat{n}} \frac{d\chi}{dk_i} + A(\hat{n}) \frac{d^2\chi}{dk_i^2} \alpha \right] = \frac{d^2c_k}{dk_i^2} \frac{\partial k_i}{\partial \hat{n}} ,
\]
which simplifies to
\[
\frac{\partial k_i}{\partial \hat{n}} = \frac{e[1-\eta] \frac{\partial A}{\partial \hat{n}} \frac{d\chi}{dk_i}}{\frac{d^2c_k}{dk_i^2} - e[1-\eta]A(\hat{n}) \frac{d^2\chi}{dk_i^2}} > 0 . \tag{A12}
\]
When $\hat{n} = 0$, no one buys the service, it doesn’t pay for the hackers to attack the service, hence $K = 0$. When $\hat{n} = 1$, all users buy the service. Since the hacker’s expected net benefit, (7), is concave in $k$, there exists $\bar{k} > 0$ that satisfies the first order condition, (8), and maximizes the expected net benefit.

Similarly, we can show that

$$\frac{\partial k_i}{\partial f} = \frac{e^{[1-\eta]} A \chi}{\frac{d^2 c_k}{dk_i^2} - e^{[1-\eta]} A(n) \frac{d^2 \chi}{dk_i^2}} < 0,$$

(A13)

and that there exists some $k_0 > 0$ such that if $f = 0$, then $k_i = k_0$, and there exists some $k_\infty > 0$ such that if $f \to \infty$, then $k_i = k_\infty$.

**Proof of Lemma 1.** By Observations 1 and 3 respectively, $f$ is increasing in $K$ and $\hat{n}$ is decreasing in $K$. Accordingly, $A(K)$ is monotonically decreasing in $K$, regardless of the user distribution $\Phi(n)$. Further, if $K = 0$, then by (2), $\chi = 0$, hence all users would choose $f(n) = 0$ and, by (3), get $B(n|K) = \nu - p$. By assumption, $\nu - p > 0$, hence, if all $k_i = 0$, then $K = 0$, and $\hat{n} = 1$, and so, $A > 0$.

With regard to hacker targeting, by Observation 4, $k_i$ is monotonically increasing in $A$. Further, if $A = 0$ (because either $\hat{n} = 0$ or $\alpha(f(n)) = 0$, for all $n$), then hackers will not target the service, $k_i = 0$.

Figure 4.3 depicts $k_i(A)$ and $A(k)$, which describe the best response functions of the hackers and users, respectively. Since the functions are continuous, they have a non-trivial intersection, say $(k^*_i, A^*)$.
Given hacker targeting $k_1^*, ..., k_Z^*$, let $K^* = k_1^* + ... + k_Z^*$, and further, let $\hat{n}(K^*)$ and $f(n \mid K^*)$ be the marginal user and user precautions respectively. Then, by (9), the conditional vulnerability

\[
A' = \int_0^{\hat{n}(K^*)} \alpha(f(n) \mid K^*) \, d\Phi(n) .
\]

Now, we claim that $A' = A^*$, and prove the claim by contradiction as follows.

(i) Suppose otherwise that $A' > A^*$. Then, referring to Figure 4.3, the function $k_i(A)$ gives the hacker’s best-response $k_i'$. Since $k_i(A)$ is monotonically increasing in $A$, we have $k_i' > k_i^*$, and so, $K' = k_1^* + ... + k_i' + ... + k_Z^* > k_1^* + ... + k_i^* + ... + k_Z^* = K^*$. Since $\hat{n}$ is decreasing in $K$ and $f(.)$ is increasing in $K$, it follows that $\hat{n}(K') < \hat{n}(K^*)$ and $f(n \mid K') > f(n \mid K^*)$, which implies that $A' < A^*$, which contradicts the original assumption.

(ii) Suppose otherwise that $A' < A^*$. Then, referring to Figure 4.3, the function $k(A)$ gives the hacker’s best-response $k'$. Since $k(A)$ is monotonically increasing in $A$, we have $k_i' < k_i^*$, and so, $K' = k_1^* + ... + k_i' + ... + k_Z^* < k_1^* + ... + k_i^* + ... + k_Z^* = K^*$. Since $\hat{n}$ is decreasing in $K$ and $f(.)$ is increasing in $K$, it follows that $\hat{n}(K') > \hat{n}(K^*)$ and $f(n \mid K') < f(n \mid K^*)$, which implies that $A' > A^*$, which contradicts the original assumption.

Therefore, we must have $A^* = A^*$. In symmetric equilibrium, $k_i^* = k^*$, $i = 1,...,Z$.

Hence, there exists a non-trivial equilibrium comprising $k^*$, $\hat{n}(k^*)$ and $f(\hat{n} \mid k^*)$. [ ]

**Proof of Proposition 1.** Expand (8) to distinguish between the precaution of end-user $n'$ denoted $f(n')$ and the precautions of all other users, $f'$,
\[ e^{[1-\eta]} \frac{dX}{dk_i} \left[ \int_{(0,\tilde{\alpha})} \alpha(f(n))d\Phi(n) + \alpha(f(n'))d\Phi(n') + \int_{(n',\tilde{\alpha})} \alpha(f(n))d\Phi(n) \right] = \frac{dc_k}{dk_i}. \quad \text{(A14)} \]

By (A14), an increase in precautions, \( f \), by all other users except \( n' \) would reduce the term in brackets, and hence induce all hackers to reduce targeting, \( \Delta k_i < 0 \), all \( i \). This would imply \( \Delta \chi < 0 \), which in (4), shifts down the left-hand side. Therefore, user \( n' \) would reduce \( f(n') \). \[ \]

**Proof of Proposition 2.** This follows directly from the proof of Table 4.2, by noting that (A16) will hold, and hence \( \partial A / \partial c \geq 0 \), if \( c \) is sufficiently high, and not hold if \( c \) is sufficiently low.

**Proof of Table 4.2**

![Figure 4A Increase in price, \( p \)](image)

**User cost of precaution, \( c \)**

By Observations 1 and 4, an increase in the user cost of precaution, \( c \), directly leads to reduced user precautions, \( f \), and service demand, \( \hat{n} \). By (9), these have mixed effects on the users’ best-response function, \( A(K) \). By (8), the increase in the user cost of precaution
has no direct effect on \( k(A) \). Accordingly, the net effect on targeting, \( k \), and conditional vulnerability, \( A \), depends on the sign of \( \frac{\partial A}{\partial c} \), which is calculated as follows,

\[
\frac{\partial A(K)}{\partial c} = \alpha(f(\hat{n})) \frac{\partial \hat{n}}{\partial c} d\Phi(\hat{n}) + \int_0^\ast \frac{d\alpha}{dc} \frac{\partial f(n)}{dc} d\Phi(n) \tag{A15}
\]

Substituting from (4) and (A1), it follows that \( \frac{\partial A}{\partial c} \geq 0 \) if and only if

\[
-\frac{1}{[v+\hat{h}]\chi} \int_0^\ast \frac{n \cdot df}{d\Phi(n)} d\Phi(n) \geq \frac{\hat{n} \alpha(\hat{n})}{c} \frac{d\Phi(\hat{n})}{dn}
\]
or

\[
c \geq \frac{[v+\hat{h}]\chi \alpha(\hat{n}) \hat{n}}{-\int_0^\ast \frac{n \cdot df}{d\Phi(n)} d\Phi(n)} \tag{A16}
\]

We analyze two cases below.

(i) \( \frac{\partial A}{\partial c} \geq 0 \). Referring to Figure 4A, an increase in \( c \) would lead to a new equilibrium, with higher targeting, \( k'_i \leq k^*_i \), higher conditional vulnerability, \( A' \leq A^* \), and hence higher effective vulnerability, \( \chi(K')A' \leq \chi(K^*)A^* \), where \( K' = k'_1 + \ldots + k'_z \) and \( K^* = k^*_1 + \ldots k^*_{i-1} + k^*_i + k^'_{i} + \ldots + k^'_{z} \). In sum, when \( \frac{\partial A}{\partial c} \geq 0 \), we must have \( \frac{dk}{dc} \geq 0 \), all \( i \), and \( \frac{dA}{dc} \geq 0 \).

With regard to the marginal user, i.e., service demand,

\[
\frac{d\hat{n}}{dc} = \frac{\partial \hat{n}}{\partial c} + \frac{\partial \hat{n}}{\partial K} \frac{dK}{dc} = \frac{\partial \hat{n}}{\partial c} + \frac{\partial \hat{n}}{\partial K} \left[ \frac{dk_1}{dc} + \ldots + \frac{dk_z}{dc} \right] \tag{A17}
\]

By Observation 3, \( \frac{\partial \hat{n}}{\partial c} < 0 \) and \( \frac{\partial \hat{n}}{\partial K} < 0 \), while from above, \( \frac{dk_i}{dc} \geq 0 \), for all \( i \). Hence, substituting in (A17), we have \( \frac{d\hat{n}}{dc} < 0 \).

Regarding the precautions, from above, \( A' \leq A^* \), hence by (9).
\[
\frac{dA}{dc} = \alpha(f(\hat{n})) \frac{d\hat{n}}{dc} d\Phi(\hat{n}) + \int_{\hat{n}}^{\infty} d\alpha \frac{df(n)}{dc} d\Phi(n) \geq 0. \tag{A18}
\]

Now, \(d\hat{n}/dc < 0\), hence, substituting in (A18), it follows that \(df/ dc < 0\).

\(\text{(ii) } \frac{\partial A}{\partial c} < 0\). Referring to Figure 4A, an increase in \(c\) would lead to a new equilibrium, with lower targeting, \(k'_i > k^*_i\), lower conditional vulnerability, \(A' > A^*\), and hence lower effective vulnerability, \(\chi(K')A' > \chi(K^*)A^*\), where
\[
K' = k'_i + ... + k'_z \quad \text{and} \quad K^* = k'_i + ... + k'_{i+1} + k^*_i + k'_{i+1} + ... + k'_z.
\]
In sum, when
\[
\frac{\partial A}{\partial c} < 0, \quad \text{we must have } \frac{dk_i}{dc} < 0, \quad \text{all } i, \text{ and } \frac{dA}{dc} < 0.
\]

With regard to user precautions,
\[
\frac{df}{dc} = \frac{\partial f}{\partial c} + \frac{\partial f}{\partial K} \frac{dK}{dc} = \frac{\partial f}{\partial c} + \frac{\partial f}{\partial K} \left[ \frac{dk_i}{dc} + ... + \frac{dk_z}{dc} \right]. \tag{A19}
\]

By Observation 1, \(\frac{\partial f}{\partial c} < 0\) and \(\frac{\partial f}{\partial K} > 0\), while from above, \(\frac{dk_i}{dc} < 0\), for all \(i\). Hence, substituting in (A19), we have \(df/ dc < 0\).

Regarding the marginal user, from above, \(A' > A^*\), hence by (9),
\[
\frac{dA}{dc} = \alpha(f(\hat{n})) \frac{d\hat{n}}{dc} d\Phi(\hat{n}) + \int_{\hat{n}}^{\infty} d\alpha \frac{df(n)}{dc} d\Phi(n) < 0. \tag{A20}
\]

Now, \(df/ dc < 0\), hence, substituting in (A20), it follows that \(d\hat{n}/ dc < 0\).

**Enforcement rate, \(\eta\), and hacking cost, \(c_k(.)\)**

First, consider the effect of an increase in enforcement, \(\eta\). By Observations 1 and 3, the increase in enforcement has no direct effect on users’ precautions or demand \(\hat{n}\). Hence, by (9), the best-response function \(A(k)\) remains unchanged. By Observation 4, the enforcement increase directly leads hackers to reduce targeting, hence their best-response function, \(k_i(A)\),
shifts to the left. Accordingly, in the new equilibrium, targeting is lower, \( k'_i > k'_i \), and the conditional vulnerability is higher, \( A' < A' \).

Since the increase in enforcement results in lower targeting, \( k_i \), hence lower hacker effectiveness, \( \chi(K) \), but higher conditional vulnerability, \( A \), the impact on the effective user vulnerability, \( \chi A \), depends on the balance of the effects on hackers and users.

With regard to user precautions,

\[
\frac{df}{d\eta} = \frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial K} \frac{dK}{d\eta}.
\]

(A21)

By (4), \( \frac{\partial f}{\partial \eta} = 0 \), by Observation 1, \( \frac{\partial f}{\partial K} > 0 \), while from above, \( dK / d\eta < 0 \). Hence, substituting in (A21), we have \( df / d\eta < 0 \).

Similarly, with regard to the marginal user, i.e., service demand,

\[
\frac{d\hat{n}}{d\eta} = \frac{\partial \hat{n}}{\partial \eta} + \frac{\partial \hat{n}}{\partial K} \frac{dK}{d\eta}.
\]

(A22)

By (5), \( \frac{\partial \hat{n}}{\partial \eta} = 0 \), by Observation 3, \( \frac{\partial \hat{n}}{\partial K} < 0 \), while from above, \( dK / d\eta < 0 \). Hence, substituting in (A22), we have \( d\hat{n} / d\eta > 0 \), which completes the proof.

The effect of an increase in the targeting cost is similar. For brevity, we omit the proof.

**Price, \( p \)**

By Observation 1, a price increase has no direct effect on user precautions, while, by Observation 3, the price increase directly reduces the demand, \( \hat{n} \). Accordingly, by (9), for \( k_i > 0 \), the best-response function \( A(k) \) shifts downward, while, by (9), for \( k_i = 0 \), \( A(0) \) does not change with \( p \). By (8), the price increase has no direct effect on \( k_i(A) \).
Figure 4A depicts the new equilibrium: the users’ best-response function shifts from $A'(K)$ downward to $A'(K')$, while the hackers’ best-response function remains unchanged. In the new equilibrium, targeting is lower, $k_i^* > k'_i$, and the conditional vulnerability is lower, $A^* > A'$. 

Given that the increase in price, $p$, leads to lower targeting, $k$, it would, by (2) result in lower hacker effectiveness, $\chi$. Thus, the effective user vulnerability, $\chi A$, decreases with price, $p$.

With regard to user precautions,

$$\frac{df}{dp} = \frac{\partial f}{\partial p} + \frac{\partial f}{\partial K} \frac{dK}{dp}. \quad (A23)$$

By (4) $\frac{\partial f}{\partial p} = 0$, by Observation 1, $\frac{\partial f}{\partial K} > 0$, while from above, $dK/dp < 0$. Hence, substituting in (A23), we have $df/dp < 0$.

Regarding the marginal user, from above, $A^* > A'$, hence, by (9),

$$\frac{dA}{dp} = \alpha(f((\hat{n}))) \frac{d\hat{n}}{dp} \frac{d\Phi(\hat{n})}{dn} + \int_0^\hat{n} \frac{d\alpha}{df} \frac{df(n)}{dp} d\Phi(n) < 0. \quad (A24)$$

From above, $df/dp < 0$, hence substituting in (A24), it follows that $d\hat{n}/dp < 0$, which completes the proof. [ ]
Proof of Proposition 3. By assumption, \( \partial A/\partial c > 0 \), hence \( dK/dc > 0 \) and \( dW/dc < 0 \). By (12) and (14), \( |dW/dc| > |dW/d\eta| \) if and only if

\[
\left[ \eta f(n) d\Phi(n) > \frac{\partial F(n)}{\partial n} \left[ \frac{\partial \hat{n}}{\partial \eta} + \frac{\partial \hat{n}}{\partial c} \right] - \left[ v + h \right] \frac{d\chi(K)}{dK} \left[ \frac{dK}{d\eta} + \frac{dK}{dc} \right] A \right],
\]

(A25)

where

\[
\frac{\partial \hat{n}}{\partial c} = \frac{\partial \hat{n}}{\partial \eta} \frac{dK}{\partial \eta},
\]

(A26)

\[
\frac{\partial \hat{n}}{\partial c} = \frac{\partial \hat{n}}{\partial c} + \frac{\partial \hat{n}}{\partial \eta} \frac{dK}{\partial \eta},
\]

(A27)

\[
\frac{\partial K}{\partial \eta} + \frac{\partial K}{\partial A} \frac{\partial A}{\partial \eta},
\]

(A28)

\[
\frac{\partial A}{\partial \eta} = \frac{\partial A}{\partial \eta} + \frac{\partial A}{\partial dK} \frac{\partial dK}{\partial \eta},
\]

(A29)

Substituting from (A6) and (A8) in (A27),

\[
\frac{\partial \hat{n}}{\partial c} = \frac{\partial \hat{n}}{\partial c} - \frac{\partial \hat{n}}{\partial \eta} \left[ v + h \right] \frac{d\chi(K)}{dK} \left[ \frac{dK}{d\eta} + \frac{dK}{dc} \right].
\]

(A30)

Further, by differentiating (8) with respect to \( c \),

\[
\frac{d k_i}{dc} = e[1-\eta] \frac{d\chi}{d k_i} \frac{dA}{dc} - e[1-\eta] A \frac{d^2 \chi}{d k_i^2}.
\]

(A31)

In symmetric equilibrium, \( k^*_i = k_i \), \( i = 1, ..., Z \), hence, by substituting from (A31),

\[
\frac{dK}{dc} = \frac{d k_1}{dc} + ... + \frac{d k_Z}{dc} = Z \frac{d k_i}{dc} = \frac{eZ[1-\eta] d\chi}{d k_i} \frac{dA}{dc} \frac{d^2 \chi}{d k_i^2} - e[1-\eta] A \frac{d^2 \chi}{d k_i^2}.
\]

(A32)

Similarly, in symmetric equilibrium, by (A11),
\[
\frac{\partial K}{\partial \eta} = \frac{\partial k}{\partial \eta} + \ldots + \frac{\partial k}{\partial \eta} = Z \frac{\partial k}{\partial \eta} = \frac{-eZ J}{A} \frac{\partial x}{\partial k}.
\] (A33)

Substituting from (A29) in (A28), and then substituting from (A33), we have

\[
\frac{dK}{d\eta} = \frac{eZ J}{A} \frac{\partial K}{\partial \eta} \frac{dZ c_k}{dk} \frac{e[1-\eta]}{A} \frac{dZ x}{dk^2}.
\] (A34)

By substituting from (A26), (A8) and (A30), the sufficient condition (A25) simplifies to

\[
p \frac{d\Phi(\hat{n})}{dn} + \int_0^nh f(n)d\Phi(n) > -[v+h] \frac{dZ(K)}{dK} \left[p \frac{d\Phi(\hat{n})}{dn} \alpha(f(\hat{n})) + A \left[\frac{dK}{dn} + \frac{dK}{dA}\right] \right].
\] (A35)

Further substituting from (A32) and (A34) in (A35), and then simplifying, we have

\[
c \left[p \frac{d\Phi(\hat{n})}{dn} + \int_0^nh f(n)d\Phi(n) \right] \left[p \frac{d\Phi(\hat{n})}{dn} \alpha(f(\hat{n})) + A \right] \left[eZA[v+h] \left[\frac{dZ c_k}{dk^2} \right]^2 \right] > \frac{c}{1-\frac{\partial K}{\partial A} \frac{\partial A}{\partial K}} [1-\eta] \frac{c}{A} \frac{dA}{dc}.
\] (A36)

Now,

\[
\frac{dA}{dc} = \frac{\partial A}{\partial c} + \frac{\partial A}{\partial K} \frac{dK}{dc} = \frac{\partial A}{\partial c} + \frac{\partial A}{\partial K} \frac{dA}{dc},
\]

which implies

\[
\frac{dA}{dc} = \frac{\frac{\partial A}{\partial c}}{1-\frac{\partial K}{\partial A} \frac{\partial A}{\partial K}}.
\] (A37)

Substituting from (5) and (A37) in (A36), then multiplying both sides by \(1/[1-\eta]\), and then substituting from (8), and simplifying, the sufficient condition simplifies to
Condition (A38) will be satisfied if the users’ benefit relative to the cost of precaution, 

\[ \frac{v - p}{c}, \]  
and the hackers’ expected enjoyment relative to targeting cost, 

\[ e[1 - \eta] \frac{dc^\chi}{dk}, \]  
are sufficiently large. [ ]