

## MA 3219 – Computability Theory

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**Assignment for 09.03.2005.** Can be corrected on request, it is not obligatory to hand the homework in.

**1. Diagonal Method.** Prove that there is a set which is neither recursively enumerable nor has a recursively enumerable complement.

**2. S-m-n Theorem.** Prove that there is a recursive function which maps every  $e$  to a program computing  $e^x$  for input  $x$ . Give the program explicitly with two inputs  $e, x$  and then apply the s-m-n theorem.

**3. Universal Function.** There is a two-place function  $\Psi$  such that  $\Psi(x, y) = \phi_x(y)$ , that is,  $\Psi(x, y)$  takes the same value as the  $x$ -th function on input  $y$  and is undefined iff the  $x$ -th function on input  $y$  is undefined. Is the set  $\{(x, y) : \psi(x, 5 \cdot y) \downarrow\}$  recursive? Prove your answer.

**4. Closure Properties.** Let  $f$  be a total and computable function. Is the set  $U = W_{f(0)} \cup W_{f(1)} \cup W_{f(2)} \cup \dots$  recursively enumerable?