MA 3219 – Computability Theory

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Assignment for 06.04.2005. Can be corrected on request, it is not obligatory to hand the homework in.

1. Many-One Reduction. Assume that $A \leq_m C$ and $B \equiv_m C$. Which of these properties are then inherited to A or to B or to both:
(a) C is recursively enumerable;
(b) C is simple;
(c) C is creative;
(d) C is semirecursive;
(e) C is recursive.
Here the set C is semirecursive iff there is a recursive linear ordering $\sqsubset$ such that whenever $x \sqsubset y$ and $x \in C$ then also $y \in C$.

2. Turing Reduction. Which two of the following sets are Turing equivalent? Prove this equivalence:
(a) $\emptyset$;
(b) $\{e : \phi_e(2345) \downarrow = 8\}$;
(c) $\{e : \phi_e(0) \uparrow \vee \phi_e(1) \uparrow\}$;
(d) $\{e : \phi_e \text{ is total}\}$;
(e) $\{e : \phi_e \text{ is infinitely often undefined}\}$.

3. Special Cases. Prove that $\emptyset$, $\mathbb{E}$ and $\mathbb{N}$ Turing equivalent but not many-one equivalent; $\mathbb{E}$ is the set of even natural numbers.

4. Classifying Reductions. Which of the following reductions are many-one reductions and Turing reductions; if a reduction is both then state that it is a many-one reductions:
(a) $x \in A \Leftrightarrow x \notin B$;
(b) $x \in A \Leftrightarrow 2x \in B \wedge 2x + 1 \notin B$;
(c) $x \in A \Leftrightarrow 256^x \in B$;
(d) $x \in A \Leftrightarrow \forall y\, (2^x \cdot 3^y \in B)$;
(e) $x \in A \Leftrightarrow y$ is even for the least y with $2^x \cdot 3^y \in B$.
At (e) it is assumed that for every x there is a y with $2^x \cdot 3^y \in B$.

5. Growth of functions. Let A, B be r.e. sets and assume that for every total A-recursive function f there is a total B-recursive function g such that $\forall x\, (f(x) \leq g(x))$. Prove that then $A \leq_T B$.