MA 3205 – Set Theory – Homework due Week 3

Frank Stephan, fstephan@comp.nus.edu.sg, 6516-2759, Room S14#04-13.

Homework. The homework follows the lecture notes. You have to hand in one starred homework in Weeks 3–6, in Weeks 7–10 and in Weeks 11–13. Further homework can be checked on request. Homework to be marked should be handed in on Tuesday of the week when it is due.

Exercise 1.5. The property of being well-founded is an abstract property which applies also to some but not all graphs which are different from the universe of all sets. Here some examples of graphs. Which of them are well-founded? The answers should be proven.

1. the set $\{0, 1, \{0\}, \{1\}, \{0, 1, \{0\}\}, \{\{1\}\}, \{\{\{1\}\}\}, 512\}$ with (a, b) being an edge iff $a \in b$;
2. the set $\{0, 1, 2, 3\}$ with the edges $(0, 1), (1, 0), (2, 3)$;
3. the set $\mathbb{N}$ of the natural numbers with every edge being of the form $(n, n + 1)$;
4. the set $\mathbb{Z}$ of the integers with the edges being the pairs $(n, n + 1)$ for all $n \in \mathbb{Z}$;
5. the set $\mathbb{Q}$ of rational numbers with the edges being the pairs $(q, 2q)$ for all $q \in \mathbb{Q}$;
6. the set $\mathbb{Q}$ of rational numbers with the edges being the pairs $(q, q + 1)$ for all $q \geq 0$ and $(q, q - 1)$ for all $q \leq 0$.

Exercise 2.5*. Determine the power-set of $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$. Is there any set X such that $\mathcal{P}(X)$ has exactly 9 elements?

Exercise 2.10. Show that every property p satisfies the following statements.

1. There are sets x, y such that $x \in y$ and either $p(x) \wedge p(y)$ or $\neg p(x) \wedge \neg p(y)$.
2. There is a set x with $x = \{y \in x \mid p(y)\}$.
3. There is a one-to-one function f such that $p(x)$ iff $p(y)$ for all $y \in f(x)$.

Exercise 2.18. Prove that the symmetric difference is associative, that is, for all sets A, B, C, $(A \triangle B) \triangle C = A \triangle (B \triangle C)$. For this reason, one can just write $A \triangle B \triangle C$. Furthermore, prove that $A - B = A \cap (A \triangle B)$.