MA 3205 – Set Theory – Homework due Week 4

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Homework. The homework follows the lecture notes. You have to hand in one starred homework in Weeks 3–6, in Weeks 7–9 and in Weeks 10–13. Further homework can be checked on request. Homework to be marked should be handed in after the lecture on Tuesday of the week when the homework is due.

Exercise 2.22. Many Boolean Algebras have a complementation operation, but in Property 2.21 only the set difference is used in the De Morgan Laws below for sets A, B, C. Why?

$$\text{De Morgan Laws:} \quad \begin{array}{rcl} A - (B \cap C) & = & (A - B) \cup (A - C), \\ A - (B \cup C) & = & (A - B) \cap (A - C). \end{array}$$

Exercise 3.12. Let $A = \{0, 1, 2\}$ and $F = \{f : A \to A \mid f = f \circ f\} = \{f : A \to A \mid \forall x\, (f(x) = f(f(x)))\}$. Show that F has exactly 10 members and determine these.

Exercise 4.6*. Which of the following sets is transitive and which is inductive?

1. $A = \{\emptyset, \{\emptyset\}\}$,
2. $B = \{\emptyset, \{\{\{\emptyset\}\}\}\}$,
3. $C = \{x \mid \forall y \in x\, \forall z \in y\, (z = \emptyset)\}$,
4. D is the closure of $\{\emptyset, \mathbb{N} \times \mathbb{N}\}$ under the successor operation $x \mapsto S(x)$,
5. E is the set of even numbers,
6. F is the set of all natural numbers which can be written down with at most 256 decimal digits,
7. G is the set of all finite subsets of $\mathbb{N}$,
8. $H = \mathcal{P}(G)$.

Exercise 4.9. Assume that a property p satisfies

$$p(1) \text{ and } \forall x\, (p(x) \Rightarrow p(S(S(x)))).$$

Show that $p(x)$ is true for all odd numbers. Assume now that one chooses the property p to identify the odd numbers, that is,

$$p(x) \Leftrightarrow \exists y \in \mathbb{N}\, (x = y + y + 1).$$

Show that p then satisfies the above condition.