MA 3205 – Set Theory – Homework due Week 10

Frank Stephan, fstephan@comp.nus.edu.sg, 6516-2759, Room S14#04-13.

Homework. The homework follows the lecture notes. You have to hand in at least three starred homeworks throughout the semester. Further homework can be checked on request. Homework to be marked should be handed in after the lecture on Tuesday of the week when the homework is due.

Exercise 13.6. Let A be some set and let $a_0a_1 \ldots a_{n-1} \, R \, b_0b_1 \ldots b_{m-1} \Leftrightarrow n < m$ and there is a function $f : n \to m$ such that $b_{f(i)} = a_i$ and $(i < j \Rightarrow f(i) < f(j))$ for all $i, j \in n$ where $a_0a_1 \ldots a_{n-1}, b_0b_1 \ldots b_{m-1} \in A^*$. Show that R is well-founded.

Let R be such that $x \, R \, y$ iff there is a z with $x \in z \wedge z \in y$. Show that R is well-founded.

Let $(x, y) \, R \, (v, w)$ iff either $x = v \wedge y \in w$ or $y = w \wedge v \in x$. Is R well-founded?

Is the relation R given as $x \, R \, y \Leftrightarrow x \cap y = x \cup y$ well-founded?

Exercise 13.11*. Construct by transfinite recursion a function on ordinals which tells whether an ordinal is even or odd. More formally, construct a function F such that $F(\alpha) = 0$ if α is even, $F(\alpha) = 1$ if α is odd. Limit ordinals should always be even; the successor of an even ordinal is odd and the successor of an odd ordinal is even.

Exercise 13.12. Is it possible to define a function F on all sets such that $F(X) = n$ iff n is the maximal number such that there are $Y_0, Y_1, \ldots, Y_n$ with $Y_{m+1} = S(Y_m)$ for all $m \in n$ and $X = Y_n$? If so, construct the corresponding function F by transfinite recursion.