MA 3205 – Set Theory – Homework due Week 12

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Homework. The homework follows the lecture notes. You have to hand in at least three starred homeworks throughout the semester. Further homework can be checked on request. Homework to be marked should be handed in after the lecture on Tuesday of the week when the homework is due.

Exercise 15.11*. Determine the Cantor Normal Form of the following ordinals.

1. $\omega + \omega^2 + \omega^3 + \omega^4 + 2$,
2. $(\omega + 3)^5 + (\omega^2 + 17) \cdot (\omega + 8) + \omega^{12}$,
3. $\omega^2 + \omega + 1 + \omega^2 + \omega + 1 + \omega^2 + \omega + 1$,
4. $1 \oplus \omega \oplus \omega^2 \oplus \omega^3$,
5. $\omega^{\omega+5} + \omega^{\omega+2} \cdot \omega + \omega^2$,
6. $256^{256} + \omega \cdot 42$.

Exercise 15.12. Assume that $\alpha = \omega^{\gamma_1} + \omega^{\gamma_2}$ and $\beta = \omega^{\delta_1} + \omega^{\delta_2}$ with $\gamma_1 > \gamma_2$ and $\delta_1 > \delta_2$. What condition on $\gamma_1, \gamma_2, \delta_1, \delta_2$ is equivalent to the equation $\alpha + \beta = \alpha \oplus \beta$.

Exercise 16.9. Construct a one-to-one function h which maps $\alpha \times \omega$ to α for any infinite limit ordinal α. This function can without loss of generality assume that the input is of the form $(\gamma \cdot \omega + n, m)$ where $m, n \in \mathbb{N}$ and γ is an ordinal with $S(\gamma) \cdot \omega \leq \alpha$; the image should be of the form $\gamma \cdot \omega + \tilde{h}(n, m)$ for some function $\tilde{h} : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$.