Department of Mathematics

Honours Project Proposal

Supervisor's info :

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This project is suitable for: Individual only

Title : Measure functions and randomness tests

Subject Classification : [6. Mathematical Logic and Theory of Computation]

Using the following list, indicate the most appropriate classification for your project in the brackets above:

1. Algebra & Number Theory
2. Analysis
3. Approximation & Wavelets
4. Combinatorics & Graph Theory
5. Differential Equations & Numerical Analysis
6. Logic & Theory of Computation
7. Mathematical Physics
8. Operations Research
9. Probability & Statistics
10. Topology & Geometry
11. Miscellaneous

Description of the scope of the project: In Algorithmic Randomness, one says that an infinite binary sequence $a_0a_1a_2\ldots$ is random iff there is no effective test covering it. A measure function ρ like $\rho(a_0a_1\ldots a_{n-1}) = 2^{-n}$ defines implicitly the measure of all sequences starting with $a_0a_1\ldots a_{n-1}$, here the two most common tests:

(a) Martin-Löf Test: This is a sequence $V_0, V_1, \ldots$ of sets of binary strings σ such that
- $\{(\sigma, n) : \sigma \in V_n\}$ is a recursively enumerable set;
- for all n, $\sum_{\sigma \in V_n} \rho(\sigma) < 2^{-n}$.

A sequence $a_0a_1a_2\ldots$ is random iff there is no Martin-Löf test such that for every n there is an m with $a_0a_1\ldots a_{m-1} \in V_n$.

(b) Solovay Test: This is a set W of binary strings such that
- W is recursively enumerable;
- $\sum_{\sigma \in W} \rho(\sigma) < \infty$.

A sequence $a_0a_1a_2\ldots$ is random iff there is no Solovay-test W such that $a_0a_1\ldots a_{n-1} \in W$ for infinitely many n.

The notions of randomness depend on the choice of ρ. For the standard choice of ρ given above, both definitions are equivalent. But one can show that for other choices like $\rho(\sigma) = 2^{-0.5 \cdot \mathrm{length}(\sigma)}$ the defined notions are different. Nevertheless, one direction survives this change: every set which is random according to all Martin-Löf tests is also random according to all Solovay tests.

The candidate should give an introduction to the field and review the current state of the art. Furthermore, the candidate should carry on the investigations which choice of axioms for a measure function is the most reasonable one. Which properties of a measure function permit to construct a set which is random according to all Solovay tests but not according to all Martin-Löf tests.

Level of difficulty:

[ ] Less Difficult [X] Moderately difficult [ ] Difficult

The supervisor's perspective of the level of difficulty in this project may not be the same as the students.

Student should clarify with the supervisor, if in doubt.

Expectations: Summarize the state of the art and the work achieved by Reimann and Stephan, investigate suitable systems of axioms for measure functions, compare the various randomness tests based on these measure functions.

Prerequisites: Module MA 3219 or good knowledge of below book from Cutland.

Relevant MA4000 modules / co-requisites: No MA4000 module required.

References:

Cristian Calude, Ludwig Staiger and Sebastiaan A. Terwijn. On partial randomness. Technical Report 239, Centre for Discrete Mathematics and Theoretical Computer Science, University of Auckland, Auckland, New Zealand, 2004.

Nigel Cutland, Computability, Cambridge University press, 1980.

Jan Reimann and Frank Stephan, On hierarchies of random tests, Manuscript, 2005.