Midterm Examination 1
MA 3205: Set Theory

15.09.2009, 12.00-12.45h

Matriculation Number: ________________

Rules
Each question contains as many marks as it has subquestion. Each correct subquestion gives 1 mark. The maximum score is 15 marks.

Question 1. Determine the following sets where $A = \{1, 2, 4, 8, 16\}$ and $B = \{3, 4, 5, 6, 7, 8\}$:
(a) $A \cup B = \{$____$\}$;
(b) $A \cap B = \{$____$\}$;
(c) $A \Delta B = \{$____$\}$.
Here $\cup$ is the union, $\cap$ the intersection and Δ the symmetric difference.

Question 2. Let A be the powerset of $\mathbb{N}$, that is, let A be the set of all subsets of $\mathbb{N}$. Check the correct box for each set.

(a) The set $\{B \in A : \mathbb{N} \subseteq B\}$ is
☐ empty ☐ finite and not empty ☐ countable ☐ uncountable.

(b) The set $\{C \in A : C \text{ has 5 elements}\}$ is
☐ empty ☐ finite and not empty ☐ countable ☐ uncountable.

(c) The set $\{D \in A : D \text{ is infinite}\}$ is
☐ empty ☐ finite and not empty ☐ countable ☐ uncountable.

Question 3. (a) Is there a set A such that A has more elements then $\cup A$?

☐ Yes; ☐ No.

(b) Write a few lines to justify your answer (no complete proof needed, but it should make sense; only counted if (a) is correct).

Question 4. (a) Is there a set B such that $B \neq \mathbb{N}$, B is transitive and B is inductive?

☐ Yes; ☐ No.

(b) Write a few lines to justify your answer (no complete proof needed, but it should make sense; only counted if (a) is correct).

Question 5. (a) Is there a set C such that the power set $\mathcal{P}(C)$ of C is countable?

☐ Yes; ☐ No.

Here recall that the statement "$\mathcal{P}(C)$ is countable" implies that "$\mathcal{P}(C)$ is infinite".

(b) Write a few lines to justify your answer (no complete proof needed, but it should make sense; only counted if (a) is correct).

Question 6. (a) Determine all sets A which satisfy $\mathcal{P}(A) \subseteq \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$:

(b) Determine all sets B which satisfy $B \subseteq \mathbb{N}$ and $\forall n[n \in B \Leftrightarrow n + 2 \in B]$:

(c) How many sets $C \in \mathbb{N}$ have at most 5 elements?

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

☐ 5 ☐ 6 ☐ 7 ☐ infinitely many.

Working Space

You can use this page to do calculations, but you should write the answers into the space provided. Answers found here are not evaluated.