CTL Model Checking

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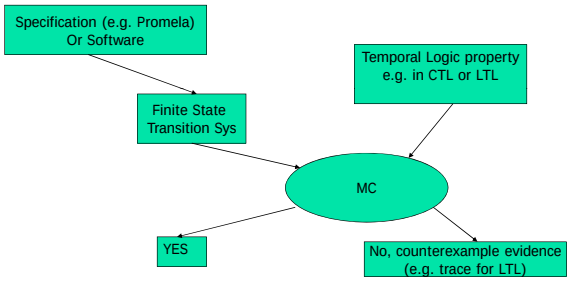
News

- Turing Award 2007 (announced Feb 08)
  - Clarke, Emerson, Sifakis
    - For their role in developing Model-Checking into a highly effective verification technology, widely adopted in the hardware and software industries.

http://awards.acm.org/homepage.cfm?srt=all&awd=140

Award in 1996 for temporal logics to Amir Pnueli

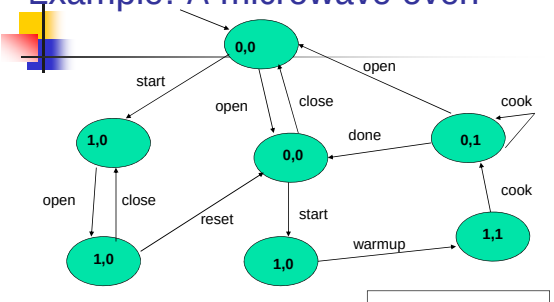
Model Checking



CTL Model Checking

- Inputs:
  - Finite state Kripke Structure M = (S,I, $\rightarrow$, L)
    - S (set of states),
    - I $\subseteq$ S (set of initial states),
    - $\rightarrow$ (transition relation),
    - L (function labeling atomic propositions to states)
  - CTL formula φ (with atomic propositions corresponding to those appearing in M)
- Output:
  - Whether for all s in I, we have M,s |= φ

Example: A microwave oven



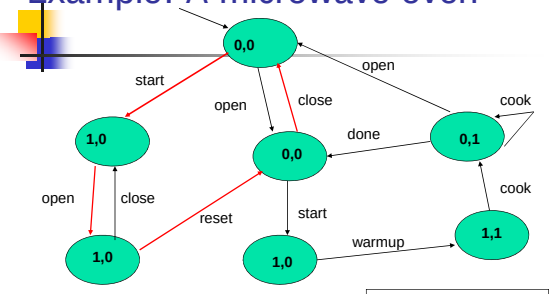
Assignment of **start**, **heat** are shown

AG (start ⇒ AF heat)

Example

- AG (start $\Rightarrow$ AF heat)
  - For any reachable state, if start holds, then along all outgoing paths, heat eventually holds.
  - Violated if:
    - $\exists$ a reachable state s where start holds
    - $\exists$ an acyclic path from s to s' in which heat does not hold in any state
    - And there is a cycle containing s' such that heat does not hold in all states of the cycle.
- Can we find such a cycle in the given model ?

Example: A microwave oven



Assignment of **start**, **heat** are shown

AG (start ⇒ AF heat)

CTL Model Checking

- A systematic way of doing the reasoning performed in our example for all CTL formulae.
- Recall set of all CTL formulae
- φ = Prop | $\neg \varphi$ | $\varphi \wedge \varphi$ | AX φ | EX φ | AF φ | EF φ
- | EG φ | AG φ | E(φ U φ) | A(φ U φ)
- | E (φ R φ) | A (φ R φ)
- Among the temporal operators, only consider
  - EX, EG, EU (along with $\neg$ and $\wedge$)

CTL temporal operators

- AX φ = $\neg\neg$AXφ = $\neg$EX $\neg\varphi$
- AGφ = $\neg\neg$AGφ = $\neg$EF $\neg\varphi$
- EFφ = E (true U φ)
- AFφ = $\neg$EG $\neg\varphi$
- A(φ R Ψ) = $\neg\neg$ A(φ R Ψ) = $\neg$ E ($\neg$ φ U $\neg$ Ψ)
- A(φ U Ψ) = $\neg$ E ($\neg$ φ R $\neg$ Ψ)
- What about E (φ RΨ) ??

Model Checking Algorithm

- Pre-processing: Rewrite the CTL formula to be verified to contain EX,EG,EU,$\wedge$, $\neg$
- For all sub-formulae x of the re-written formula φ, find the set of states satisfying x in the given model M.
- The above step finally computes the set of states in M which satisfy φ, call it St_{φ}.
- Check whether all initial states of M are contained in St_{φ}.

Word of caution

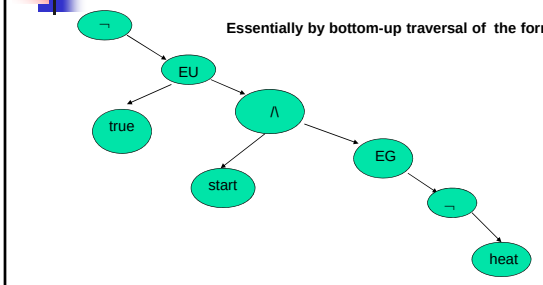
- The pre-processing of the formula is only being done to simplify the presentation of the MC algorithm
  - Reduce number of cases to consider.
- We can
  - Develop customized algorithms for each of the 10 CTL operators, and then
  - apply them in a bottom-up recursive fashion as we will be doing now.

Example

- M = the model of microwave oven given earlier
- φ = AG (start $\Rightarrow$ AF heat)
- = $\neg$ EF $\neg$($\neg$start $\vee$ AF heat)
- = $\neg$ EF (start $\wedge$ $\neg$ AF heat)
- = $\neg$ EF (start $\wedge$ EG $\neg$ heat)
- = $\neg$ E(true U (start $\wedge$ EG $\neg$ heat))
- Now, how to compute the set of states in M satisfying this transformed formula ?

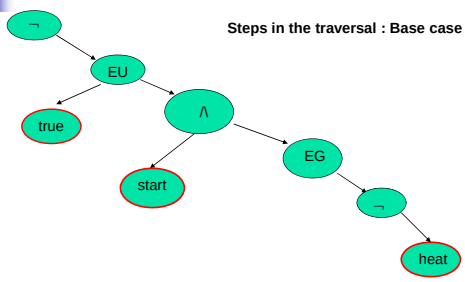
Example

Essentially by bottom-up traversal of the formula



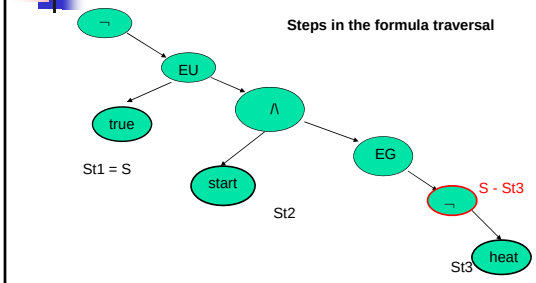
Example

Steps in the traversal : Base case



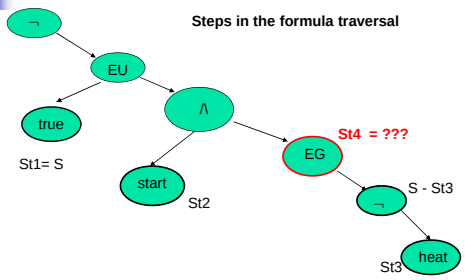
Example

Steps in the formula traversal



Example

Steps in the formula traversal



Example

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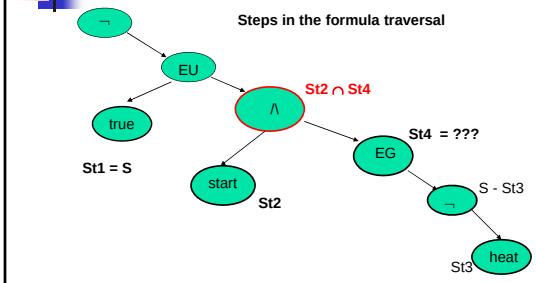
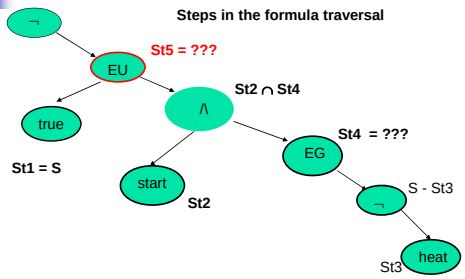
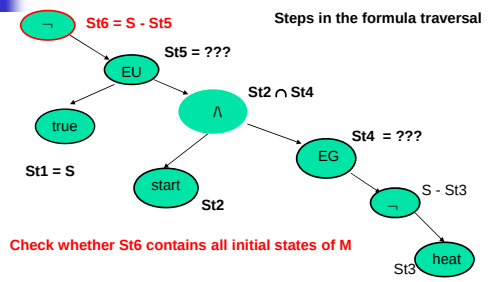


Example

Steps in the formula traversal



Example



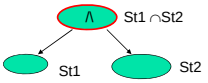
Check whether St6 contains all initial states of M

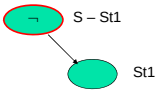
Bottom-up formula traversal

Base case: Atomic propositions

The set of states satisfying p in M = (S,S0,R,L), is found from L, the labeling function of M

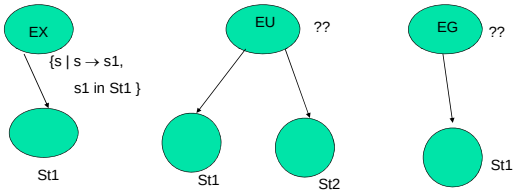
Boolean operators: ∧, ¬





Questions Remaining

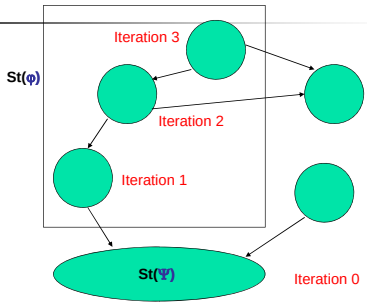
Temporal operators: EX, EU, EG



EU

- Inputs:
  - Kripke Structure M = (S, I, →, L).
  - CTL formulae φ and Ψ.
  - St_φ, set of states satisfying φ in M.
  - St_Ψ, set of states satisfying Ψ in M.
- Output:
  - Set of states satisfying E(φ U Ψ) in M.
- Technique:
  - ***Traversing the states (and transitions) of M.***

E(φ U Ψ) : Intuition



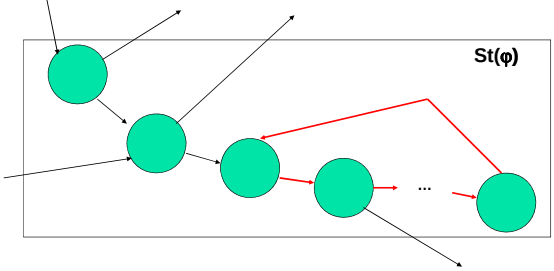
E(φ U Ψ) : Algorithm

- Result := St_Ψ ;
- Temp := St_Ψ ;
- while Temp ≠ empty do
- pick s ∈ Temp; Temp := Temp – {s};
- Backstep:= {s1 | s1 → s, and s1 ∈ St_φ };
- Temp := Temp ∪ Backstep;
- Result := Result ∪ Backstep;
- endwhile;
- return Result;
-

EG

- Inputs:
  - Kripke Structure M = (S, I, →, L).
  - CTL formulae φ.
  - St_φ, set of states satisfying φ in M.
- Output:
  - Set of states satisfying EG φ in M.
- Technique:
  - ***Traversing the states (and transitions) of M.***

EG φ : Intuition



EG φ : Algorithm

- Result := St_φ;
- **repeat**
- Temp := { s | s ∈ Result , and
- ∀s1. s → s1 ⇒ s1 ∉ Result } ;
- Result := Result – Temp;
- **until** Temp = empty;
- **return** Result;

How to make it more efficient

- We initialize $St_{EG\varphi} = St_\varphi$
  - For each state in St_φ, we check the out-edges. Many of the destination states are not in St_φ, so cannot satisfy EGφ
- It suffices to consider a reduced Kripke Structure M' constructed from M such that
  - All states of M which satisfy φ are retained.
  - All other states and transitions are deleted.
- For any s, we have M,s |= EGφ if and only if
  - s is a state in M'
  - s reaches a state s' in M' where s' loops back to itself.

Efficient computation

- Input: M = (S,I,→,L), St_φ
- Output: $St_{EG\varphi}$
- Technique:
  - Compute M'=(S',I',→',L') from M by keeping only nodes in St_φ
  - Temp := $St_{EG\varphi}$:= All nodes in nontrivial SCCs of M'
  - while Temp ≠ empty do
  - pick s ∈ Temp; Temp := Temp – {s};
  - $St_{EG\varphi}$:= $St_{EG\varphi}$ ∪ { t | t →' s ∧ t ∉ $St_{EG\varphi}$ };
  - Temp := Temp ∪ { t | t →' s ∧ t ∉ $St_{EG\varphi}$};
  - endwhile

SCC = Strongly connected component = Maximal sub-graph where every node has a path to every other node.

Summary, Exercises

- We have only presented model checking as a decision procedure.
- Other issues such as counter-example computation not shown.
- Direct iterative algorithms given only for EU, EG
  - What about EF, AF, AG etc. ?
- Algorithmic complexity of the iterative algorithms discussed in today's lecture.