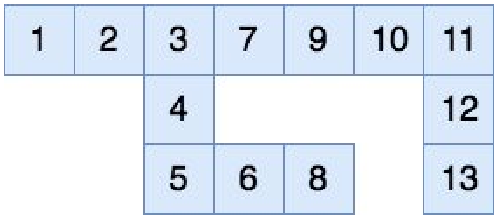
Definition

Consider the following board with 13 blocks.



Since the blocks are guaranteed to form a tree, for each block u, there is a single path from u to block number 1 -- the root of the tree. We call this path $P(u)$.

e.g.

- $P(7) = (7,\ 3,\ 2,\ 1)$
- $P(8) = (8,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1)$

For any $u \neq 1$ let's denote by $par(u)$ the block after u in $P(u)$, e.g. $par(8) = 6,\ par(7) = 3,\ par(5) = 4$. Note that $par(u)$ is uniquely defined since $P(u)$ is unique.

Observations

We can prove that each block u must be slide after $par(u)$. More precisely, the block u must be slide so that it bumps into $par(u)$ and stops. Thus, for each block we also know the direction which it is slided.

Consider block 8 in the above figure. We know that this block is slided from the right until it bumps into block 6. This also means that block 13 must be slided after block 8 (otherwise block 8 will stop when it reaches block 13).

So for each block u, we have the following 2 conditions:

- Block $par(u)$ must be slided before block u.
- Find the direction which block u is slided in. All blocks in this direction must be slided after u.

We can prove that when these 2 conditions are met for all block u, it is possible to make the target board.

Proof

Consider a sequence of blocks S, such that in this sequence:

- $par(u)$ appears before u
- All blocks in the direction which we slide block u must appear after u in sequence S.

We slide each block one by one in the order it appears in S. For each block u, we must be able to slide it so that it bumps into $par(u)$ and stops (since $par(u)$ must already be slided, and no other block is on its direction).

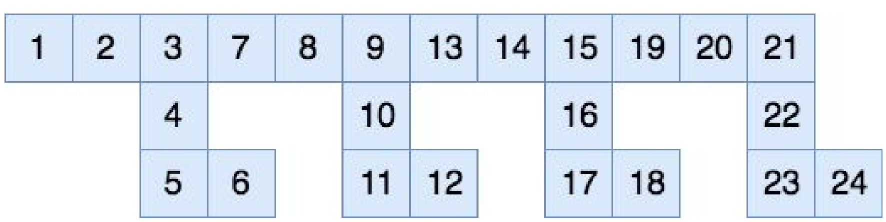
Thus, it is possible to make target board iff the 2 conditions are met for all block u, or equivalently we can find sequence S as described above.

Algorithm

1. DFS on the blocks to build the tree. During this step, we also:
   - Find $par(u)$ for each block u.
   - Find the direction which block u must be slided.
   - Find the next block in direction which u must be slided.
2. Build a new graph G, where:
   - Vertices are the blocks.
   - For each condition: u must be slided before v, add an edge (u, v).
3. Run topo sort on G to find the sequence S.

Note:

- If we add all edges of G, there would be $O(N^2)$ edges, where N is the number of blocks.



e.g. In the above figure, we need to add the following edges:

- (6, 11), (6, 12), (6, 17), (6, 18), (6, 23), (6, 24)
- (11, 12)
- (12, 17), (12, 18), (12, 23), (12, 24)
- (17, 18)
- (18, 23), (18, 24)
- (23, 24)

To reduce the number of edges to $O(N)$, notice that we only need to add edges from block u to the next block in that direction. Thus we only need $O(N)$ edges:

- (6, 11)
- (11, 12)
- (12, 17)
- (17, 18)
- (18, 23)