Definition

Consider the following board with 13 blocks.

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Since the blocks are guaranteed to form a tree, for each block $u$, there is a single path from $u$ to block number 1 -- the root of the tree. We call this path $P(u)$. e.g.
- $P(7) = (7, 3, 2, 1)$
- $P(8) = (8, 6, 5, 4, 3, 2, 1)$

For any $u \neq 1$ let's denote by $par(u)$ the block after $u$ in $P(u)$, e.g. $par(8) = 6, par(7) = 3, par(5) = 4$. Note that $par(u)$ is uniquely defined since $P(u)$ is unique.

Observations

We can prove that each block $u$ must be slide after $par(u)$. More precisely, the block $u$ must be slide so that it bumps into $par(u)$ and stops. Thus, for each block we also know the direction which it is slide.

Consider block 8 in the above figure. We know that this block is slide from the right until it bumps into block 6. This also means that block 13 must be slide after block 8 (otherwise block 8 will stop when it reaches block 13).

So for each block $u$, we have the following 2 conditions:
- Block $par(u)$ must be slide before block $u$.
- Find the direction which block $u$ is slide in. All blocks in this direction must be slide after $u$.

We can prove that when these 2 conditions are met for all block $u$, it is possible to make the target board.

Proof

Consider a sequence of blocks $S$, such that in this sequence:
• \( \text{par}(u) \) appears before \( u \)
• All blocks in the direction which we slide block \( u \) must appear after \( u \) in sequence \( S \).

We slide each block one by one in the order it appears in \( S \). For each block \( u \), we must be able to slide it so that it bumps into \( \text{par}(u) \) and stops (since \( \text{par}(u) \) must already be slided, and no other block is on its direction).

Thus, it is possible to make target board iff the 2 conditions are met for all block \( u \), or equivalently we can find sequence \( S \) as described above.

Algorithm

1. DFS on the blocks to build the tree. During this step, we also:
   - Find \( \text{par}(u) \) for each block \( u \).
   - Find the direction which block \( u \) must be slided.
   - Find the next block in direction which \( u \) must be slided.

2. Build a new graph \( G \), where:
   - Vertices are the blocks.
   - For each condition: \( u \) must be slided before \( v \), add an edge \((u, v)\).

3. Run topo sort on \( G \) to find the sequence \( S \).

Note:
• If we add all edges of \( G \), there would be \( O(N^2) \) edges, where \( N \) is the number of blocks.

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e.g. In the above figure, we need to add the following edges:
• \((6, 11), (6, 12), (6, 17), (6, 18), (6, 23), (6, 24)\)
• \((11, 12)\)
• \((12, 17), (12, 18), (12, 23), (12, 24)\)
• \((17, 18)\)
• \((18, 23), (18, 24)\)
• \((23, 24)\)

To reduce the number of edges to \( O(N) \), notice that we only need to add edges from block \( u \) to the next block in that direction. Thus we only need \( O(N) \) edges: