

CS2030S Recitation

Problem Set 8

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Recap: Functors

Recap: Functors

- A functor is a *structure* with 2 methods (`of`, `map`) obeying two laws:
 1. Identity morphism (mapping identity function gives you the same functor)
 - ▶ \forall functor: `functor.map(x \rightarrow x)` \equiv functor
 2. Composition morphism (For ever 2 maps, there's a map that composes the 2)
 - ▶ \forall functor, `f,g`: `functor.map(x \rightarrow f(x)).map(y \rightarrow g(y))` \equiv `functor.map(x \rightarrow g(f(x)))`

Recap: Monads

Recap: Monads

- Simple definition
- A Monad in X is a monoid in the category of endofunctors of X

Question 1a: Background

```
1 class Monad<T> {
2     private T x;
3     private Monad(T x) { this.x = x; }
4
5     public static <T> Monad<T> of (T x) { return new Monad<>(x); }
6
7     public T get() { return x; }
8
9     public <R> Monad<R> flatMap(Transformer<? super T,
10         ? extends Monad<? extends R>> f) {
11         return new Monad<>(f.transform(this.x).get());
12     }
13
14     public <R> Monad<R> map(Transformer<? super T, ? extends R> f) {
15         return this.flatMap(??); // <-- What to do here?
16     }
17 }
```

Recap: Monads

- A box with context (side information)
- Has a `flatMap` method (takes in a function $f: X \rightarrow \text{Monad}\langle X \rangle$, composes contexts)
- Has a `of` method (Wrap X into the `Monad<X>`)
- Has to fulfill 3 laws
 1. Left identity (`of` should not add effects/context)
 - ▶ $\forall x, f: \text{Monad.of}(x).flatMap(y \rightarrow f(y)) \equiv f(x)$
 2. Right identity (Composing the effects of `of` should not change the context)
 - ▶ $\forall m: m.flatMap(x \rightarrow \text{Monad.of}(x)) \equiv \text{monad}$
 3. Associative
 - ▶ $\forall m, f, g: m.flatMap(x \rightarrow f(x)).flatMap(y \rightarrow g(y)) \equiv m.flatMap(x \rightarrow f(x).flatMap(y \rightarrow g(y)))$

Question 1a

```
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```

Question 1a



Question 1a

```
1 class Monad<T> {
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10         ? extends Monad<? extends R>> f) {
11         return new Monad<>(f.transform(this.x).get());
12     }
13
14     public <R> Monad<R> map(Transformer<? super T, ? extends R> f) {
15         return this.flatMap(x -> Monad.of(f.transform(x)))
16     }
17 }
```

Question 1b

Show that the monad preserves composition

- Show that
 - ▶ $m.\text{map}(x \rightarrow f(x)).\text{map}(y \rightarrow g(y)) \equiv m.\text{map}(x \rightarrow g(f(x)))$

Question 1b

`m.map(x → f(x)).map(y → g(y))` (Starting)

`m.flatMap(x → Monad.of(f(x))).flatMap(y → Monad.of(g(y)))` (by β -reduction/
substitution)

`m.flatMap(x → Monad.of(f(x)).flatMap(y → Monad.of(g(y))))` (by
Associativity)

`m.flatMap(x → Monad.of(g(f(x))))` (by Left Identity)

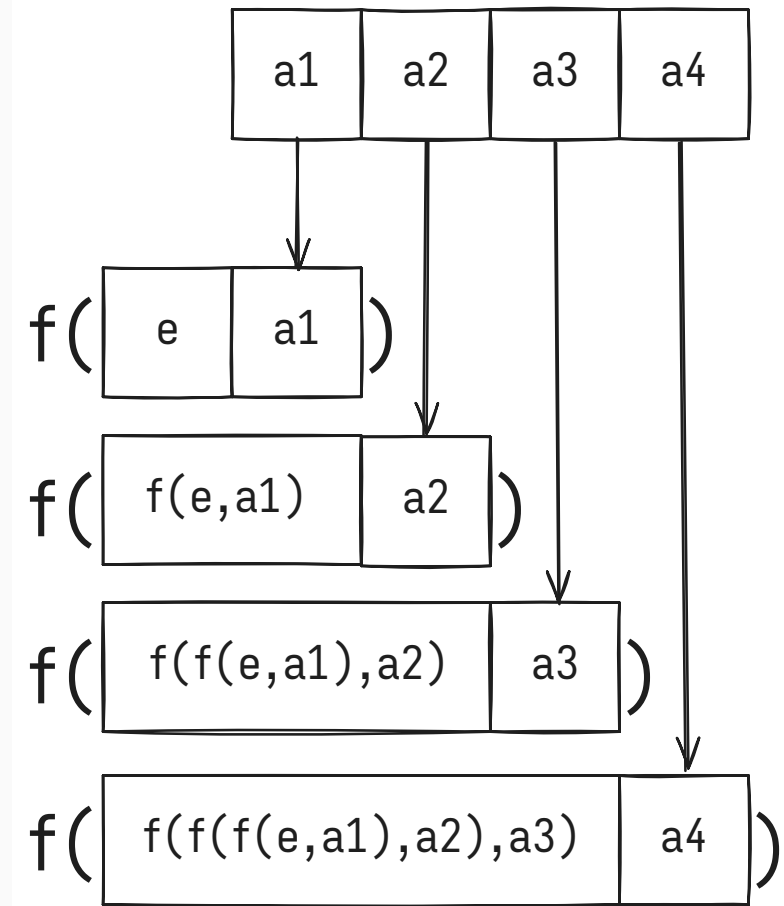
`m.map(x → g(f(x)))` (by Implementation)

Recap: Concurrency

- Sequential
 - Do things in order on one thread
- Concurrent
 - One thread does multiple things by context switching very quickly
- Parallel
 - Actually do multiple things at the same time

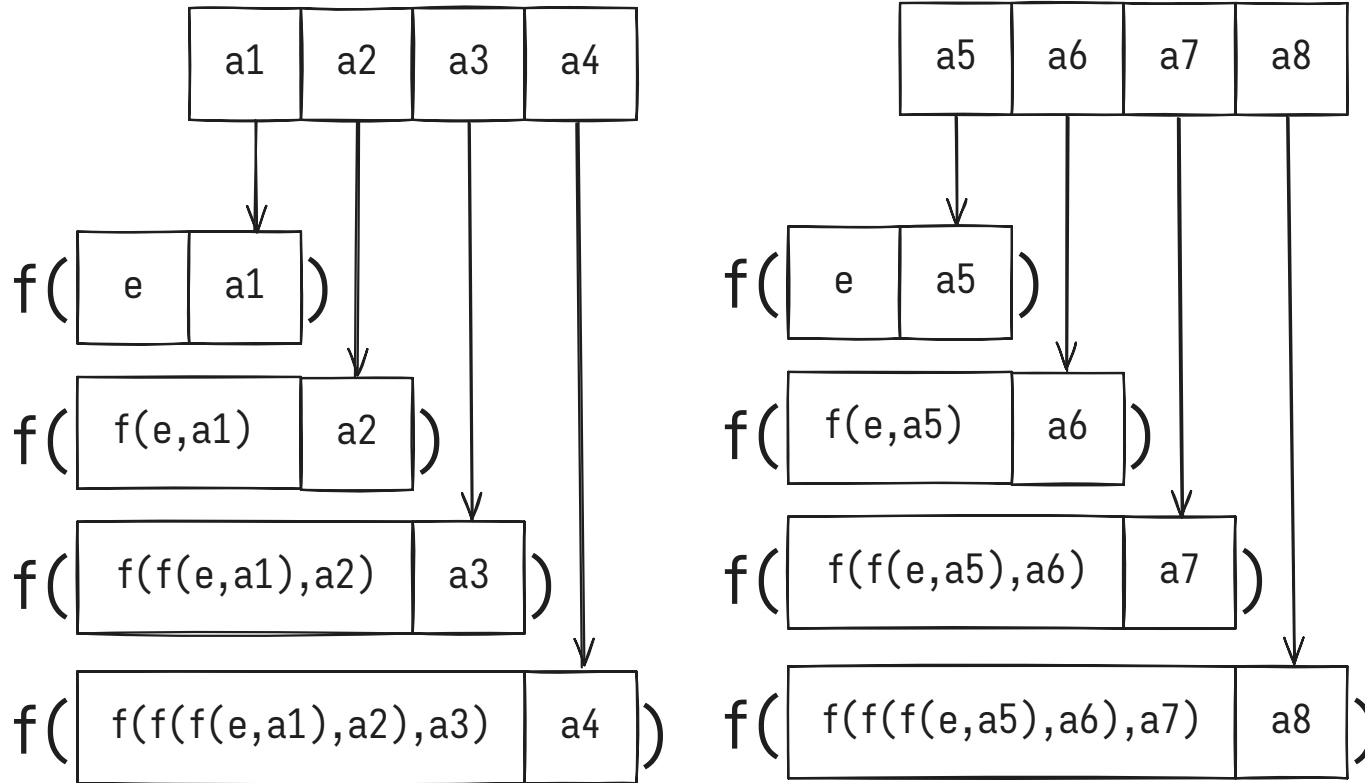
Recap: Reduce Sequential

`T reduce(T e, BinaryOperator<T> f)`



Recap: Reduce Parallel

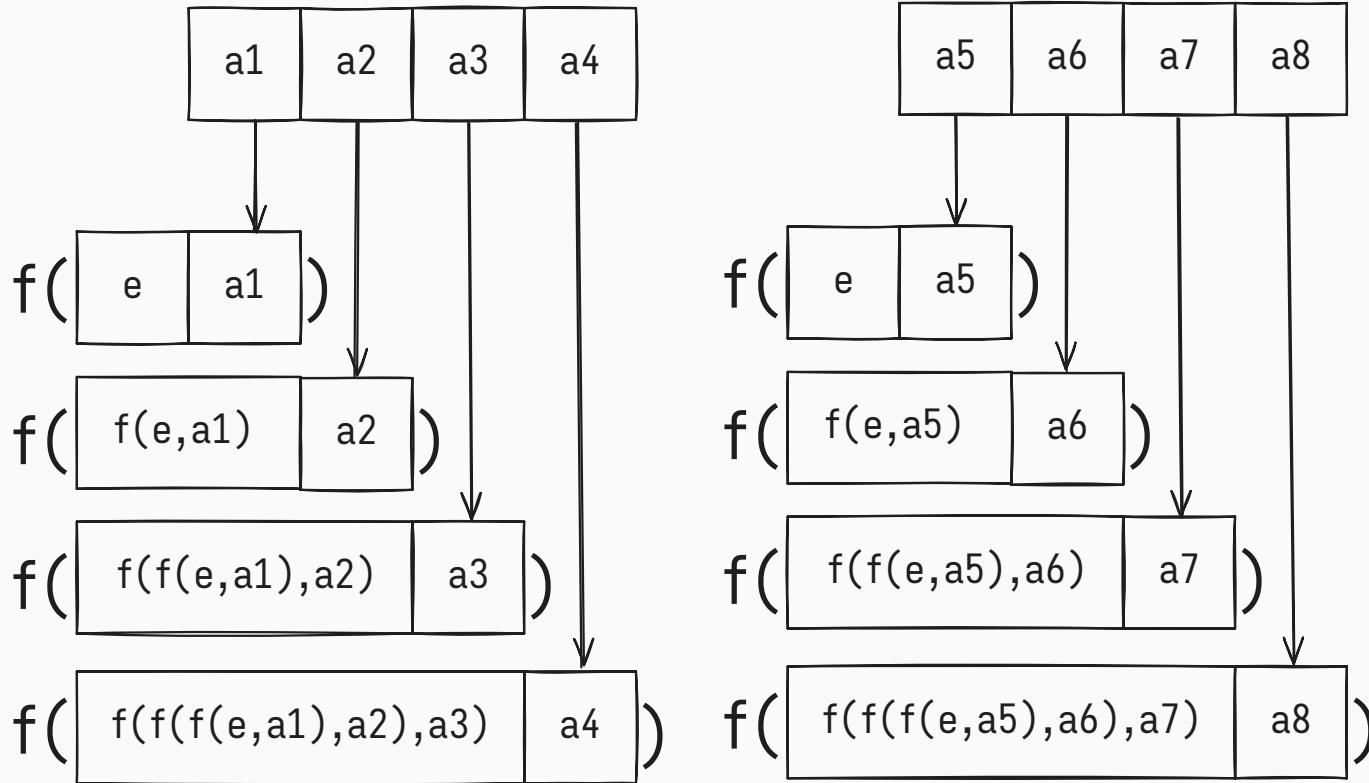
`T reduce(T e, BinaryOperator<T> f)`



`f(f(f(f(f(e, a1), a2), a3), a4) , f(f(f(f(e, a5), a6), a7), a8))`

Recap: Reduce Parallel

`<U> U reduce(U e, BiFunction<U,? super T,U> f, BinaryOperator<U> g)`



`g(f(f(f(f(e, a1), a2), a3), a4) , f(f(f(f(e, a5), a6), a7), a8))`

Question 2a and 2b

What is the returned value for each of the pipelines

1.

```
1 Stream.of(1,2,3,4)
2     .reduce(0, (a, x) -> (2 * a) + x, (a1, a2) -> a1 + a2);
```

2.

```
1 Stream.of(1,2,3,4)
2     .parallel()
3     .reduce(0, (a, x) -> (2 * a) + x, (a1, a2) -> a1 + a2);
```

Question 2a and 2b

So why is it so different?

- brian shows on whiteboard execution
- Accumulator is not associative $f(f(a, b), c) \neq f(a, f(b, c))$
- brian shows why not associative on whiteboard again
- Combiner and accumulator are also not compatible $\text{comb}(u, \text{acc}(\text{id}, t)) \neq \text{acc}(u, t)$

Question 3

Recall `estimatePi` from Lab 0

```
1 public static double estimatePi(int numOfPoints, int seed) {
2     RandomPoint.setSeed(seed);
3     Circle c = new Circle(new Point(0.5, 0.5), 0.5);
4     int n = 0;
5     for (int i = 0; i < numOfPoints; i++) {
6         Point p = new RandomPoint(0, 1, 0, 1);
7         if (c.contains(p)) {
8             System.out.println(p);
9             n++;
10        }
11    }
12    return 4.0 * n / numOfPoints;
13 }
```

What can we say about the probability of a random point being in the circle?

Question 3



Question 3

- Does parallelization make it faster?
 - ▶ No it's costly to make new threads
 - ▶ Just creating objects is fast
 - ▶ Parallelization is good if each task is very costly (in this case it's not)

Why is the number different

- Each thread has access to the same random seed
- The order which the threads interleave is random
- When limit happens, the threads may have produced more than necessary elements and would be chopped fulfill
- So what's left is a stream that can have different random points each run due to the randomness
- anyway doesn't matter don't think too much

bye!

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