CS1020 Data Structures and Algorithms I Lecture Note #14

Sorting

Objectives



- To analyse the running time of these algorithms
- To learn concepts such as in-place sorts and stable sorts
- Using Java methods to perform sorting

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References

Book

• Chapter 10: Algorithm Efficiency and Sorting, pages 542 to 577.

Data Absraction & Problem Solving with JAVA

CS1020 website \rightarrow Resources \rightarrow Lectures

http://www.comp.nus.edu.sg/ ~cs1020/2_resources/lectures.html

Programs used in this lecture

- SelectionSort.java
- BubbleSort.java, BubbleSortImproved.java
- InsertionSort.java
- MergeSort.java
- QuickSort.java
- Sort.java, Sort2.java
- Person.java, AgeComparator.java, NameComparator.java, TestComparator.java

Why Study Sorting?

 When an input is sorted by some sort key, many problems become easy (eg. searching, min, max, kth smallest, etc.)

Q: What is a sort key?

- Sorting has a variety of interesting algorithmic solutions, which embody many ideas:
 - Internal sort vs external sort
 - Iterative vs recursive
 - Comparison vs non-comparison based
 - Divide-and-conquer
 - Best/worst/average case bounds

Sorting applications

- Uniqueness testing
- Deleting duplicates
- Frequency counting
- Set intersection/union/difference
- Efficient searching
- Dictionary
- Telephone/street directory
- Index of book
- Author index of conference proceedings
- etc.

Outline

- Comparison based and Iterative algorithms
- 1. Selection Sort
- 2. Bubble Sort
- 3. Insertion Sort
- Comparison based and Recursive algorithms
- 4. Merge Sort
- 5. Quick Sort
- Non-comparison based
- 6. Radix Sort
- 7. Comparison of Sort Algorithms
 - In-place sort
 - Stable sort
- 8. Use of Java Sort Methods

Note: We consider only sorting in ascending order of data.

1 Selection Sort

1 Idea of Selection Sort

- Given an array of n items
 - 1. Find the largest item.
 - 2. Swap it with the item at the end of the array.
 - 3. Go to step 1 by excluding the largest item from the array.

1 Selection Sort of 5 integers

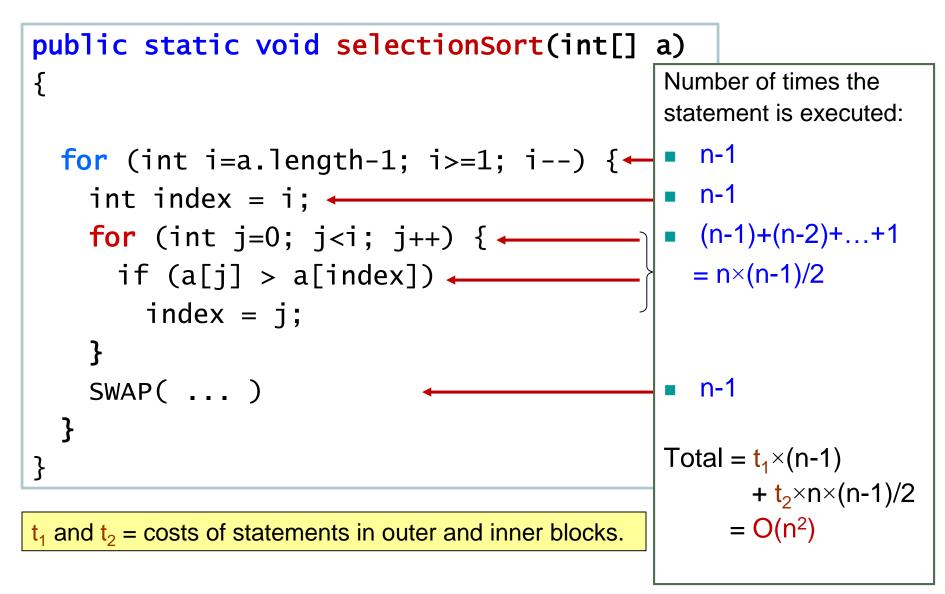
29	10	14	37	13	37 is the largthe last elemQ: How to fir
29	10	14	13	37	
13	10	14	29	37	
13	10	14	29	37	
10	13	14	29	37	Sorted!

37 is the largest, swap it with the last element, i.e. 13.Q: How to find the largest?

1 Code of Selection Sort

```
public static void selectionSort(int[] a) {
  for (int i = a.length-1; i >= 1; i--) {
    int index = i; // i is the last item position and
                     // index is the largest element position
    // loop to get the largest element
    for (int j = 0; j < i; j++) {
      if (a[j] > a[index])
         index = j; // j is the current largest item
    // Swap the largest item a[index] with the last item a[i]
    int temp = a[index];
    a[index] = a[i];
    a[i] = temp;
}
                                             SelectionSort.java
```

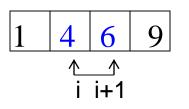
1 Analysis of Selection Sort

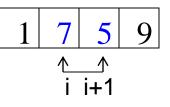


2 Bubble Sort

2 Idea of Bubble Sort

- "Bubble" down the largest item to the end of the array in each iteration by examining the i-th and (i+1)-th items
- If their values are not in the correct order, i.e.
 a[i] > a[i+1], swap them.



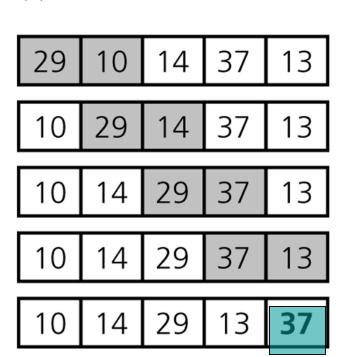


// no need to swap

// not in order, need to swap

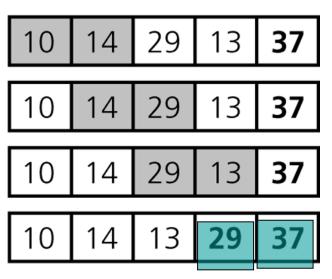
2 Example of Bubble Sort

The first two passes of Bubble Sort for an array of 5 integers



At the end of **pass 1**, the largest item **37** is at the last position.

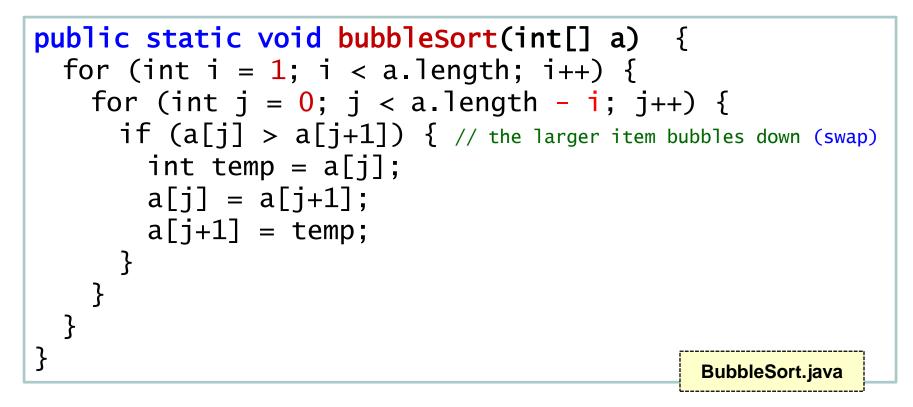
(b) Pass 2



At the end of **pass 2**, the second largest item **29** is at the second last position.

(a) Pass 1

2 Code of Bubble Sort



Bubble Sort animation

2 Analysis of Bubble Sort

- 1 iteration of the inner loop (test and swap) requires time bounded by a constant c
- Doubly nested loops:
 - Outer loop: exactly n-1 iterations
 - Inner loop:
 - When i=1, (n-1) iterations
 - When i=2, (n-2) iterations
 - ...
 - When i=(n-1), 1 iteration

- Total number of iterations = (n-1) + (n-2) + ... + 1= $n \times (n-1)/2$
- Total time = c × n × (n-1)/2 = O(n²)

2 Bubble Sort is inefficient

- Given a sorted input, Bubble Sort still requires O(n²) to sort.
- It does not make an effort to check whether the input has been sorted.
- Thus it can be improved by using a flag, isSorted, as follows (next slide):

2 Code of Bubble Sort (Improved version)

```
public static void bubbleSort2(int[] a) {
  for (int i = 1; i < a.length; i++) {
    boolean isSorted = true; // issorted = true if a[] is sorted
    for (int j = 0; j < a.length-i; j++) {
      if (a[j] > a[j+1]) \{ // \text{ the larger item bubbles up}
        int temp = a[j]; // and issorted is set to false,
        a[j] = a[j+1]; // i.e. the data was not sorted
        a[j+1] = temp;
        isSorted = false;
      }
    if (isSorted) return; // Why?
  }
}
                                            BubbleSortImproved.java
```

2 Analysis of Bubble Sort (Improved version)

Worst case

- Input in descending order
- How many iterations in the outer loop are needed? Answer: n-1 iterations
- Running time remains the same: O(n²)

Best case

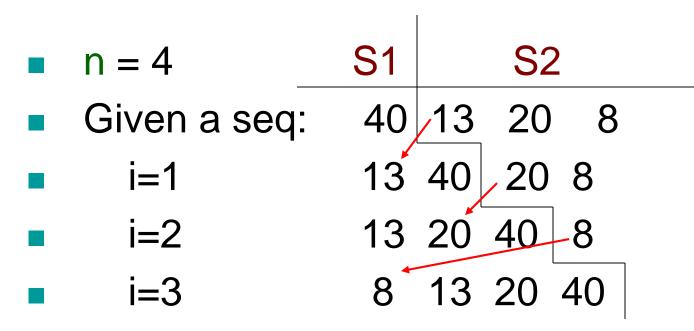
- Input is already in ascending order
- The algorithm returns after a single iteration in the outer loop. (Why?)
- Running time: O(n)

3 Insertion Sort

3 Idea of Insertion Sort

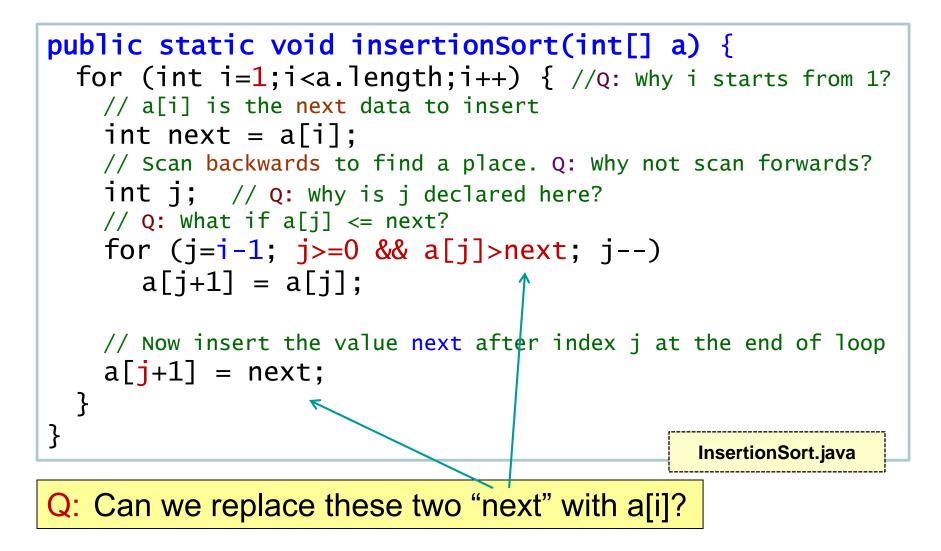
- Arranging a hand of poker cards
 - □ Start with one card in your hand
 - Pick the next card and insert it into its proper sorted order
 - Repeat previous step for all the rest of the cards

3 Example of Insertion Sort



- n = no of items to be sorted
- S1 = sub-array sorted so far
- S2 = elements yet to be processed
- In each iteration, how to insert the next element into S1 efficiently?

3 Code of Insertion Sort



3 Analysis of Insertion Sort

- Outer loop executes exactly n-1 times
- Number of times inner loop executes depends on the inputs:
 - Best case: array already sorted, hence (a[j] > next) is always false
 - No shifting of data is necessary; Inner loop not executed at all.
 - Worst case: array reversely sorted, hence (a[j] > next) is always true
 - Need i shifts for i = 1 to n-1.
 - Insertion always occurs at the front.
- Therefore, the best case running time is O(n).
 (Why?)
- The worst case running time is O(n²). (Why?)

```
... insertionSort(int[] a) {
  for (int i=1;i<a.length;i++) {
    int next = a[i];
    int j;
    for (j=i-1; j>=0 && a[j]>next; j--)
        a[j+1] = a[j];
    a[j+1] = next;
  }
}
```

4 Merge Sort

4 Idea of Merge Sort (1/3)

- Suppose we only know how to merge two sorted lists of elements into one combined list
- Given an unsorted list of n elements
- Since each element is a sorted list, we can repeatedly...
 - Merge each pair of lists, each list containing one element, into a sorted list of 2 elements.
 - Merge each pair of sorted lists of 2 elements into a sorted list of 4 elements.
 - • •
 - The final step merges 2 sorted lists of n/2 elements to obtain a sorted list of n elements.

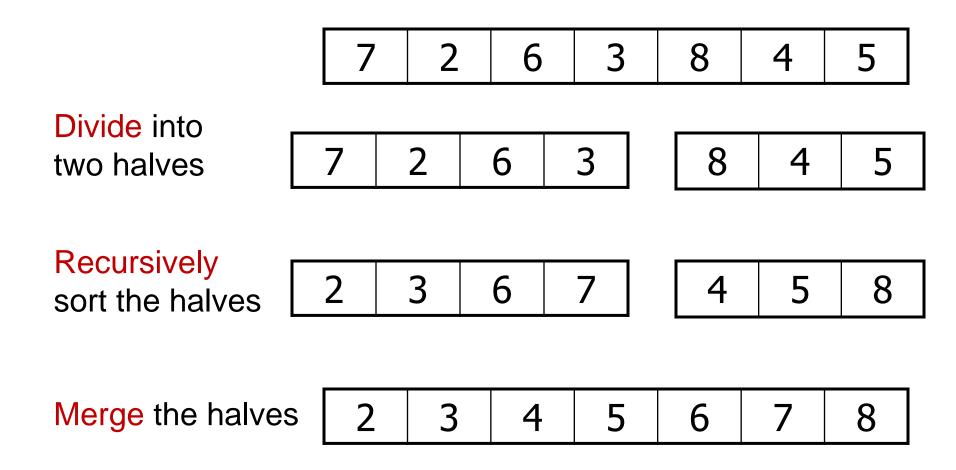
4 Idea of Merge Sort (2/3)

- Divide-and-conquer method solves problem by three steps:
 - Divide Step: divide the larger problem into smaller problems.
 - (Recursively) solve the smaller problems.
 - Conquer Step: combine the results of the smaller problems to produce the result of the larger problem.

4 Idea of Merge Sort (3/3)

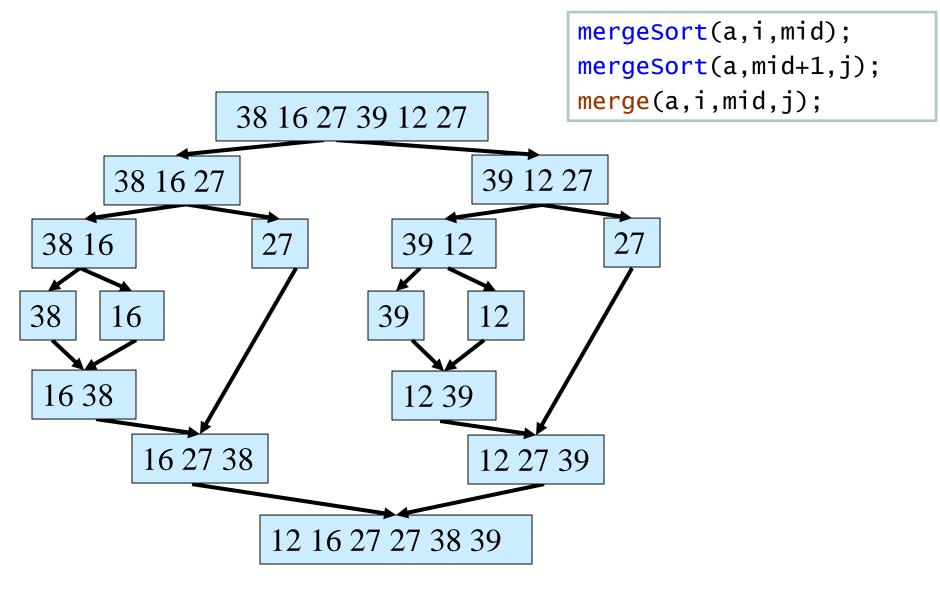
- Merge Sort is a divide-and-conquer sorting algorithm
 - Divide Step: Divide the array into two (equal) halves.
 - Recursively) sort the two halves.
 - Conquer Step: Merge the two sorted halves to form a sorted array.
- Q: What are the base cases?

4 Example of Merge Sort

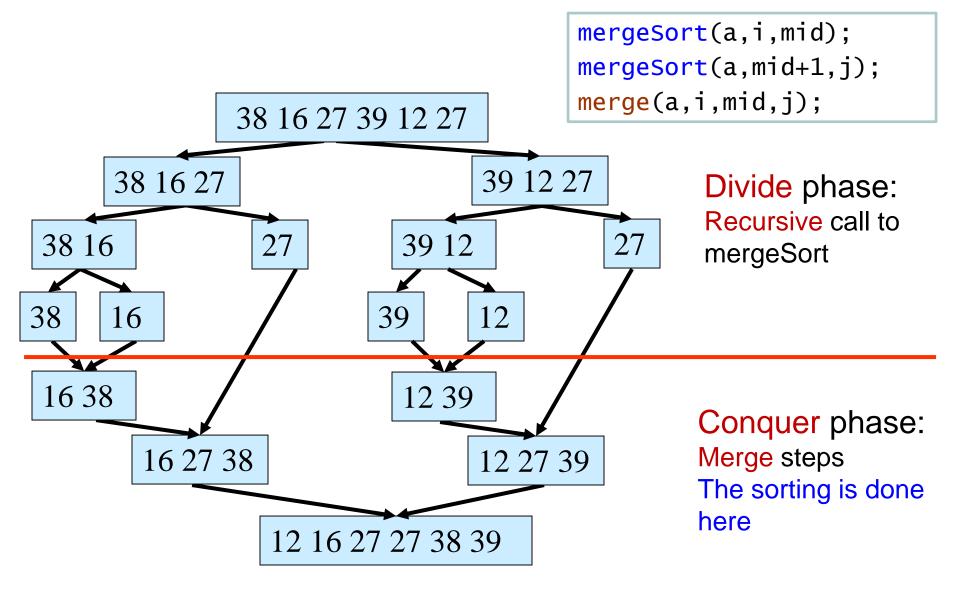


4 Code of Merge Sort

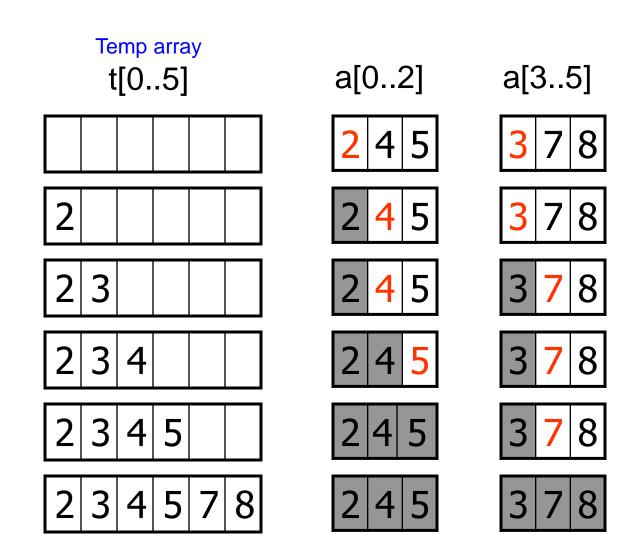
4 Merge Sort of a 6-element Array (1/2)



4 Merge Sort of a 6-element Array (2/2)



4 How to Merge 2 Sorted Subarrays?



4 Merge Algorithm (1/2)

... merge(int[] a, int i, int mid, int j) {
 // Merges the 2 sorted sub-arrays a[i..mid] and
 // a[mid+1..j] into one sorted sub-array a[i..j]

```
int[] temp = new int[j-i+1]; // temp storage
int left = i, right = mid+1, it = 0;
// it = next index to store merged item in temp[]
// Q: What are left and right?
```

```
while (left<=mid && right<=j) { // output the smaller
  if (a[left] <= a[right])
    temp[it++] = a[left++];
  else
    temp[it++] = a[right++];
}
```

4 Merge Algorithm (2/2)

```
// Copy the remaining elements into temp. Q: Why?
while (left<=mid) temp[it++] = a[left++];
while (right<=j) temp[it++] = a[right++];
// Q: will both the above while statements be executed?
// Copy the result in temp back into
// the original array a
for (int k = 0; k < temp.length; k++)
a[i+k] = temp[k];</pre>
```

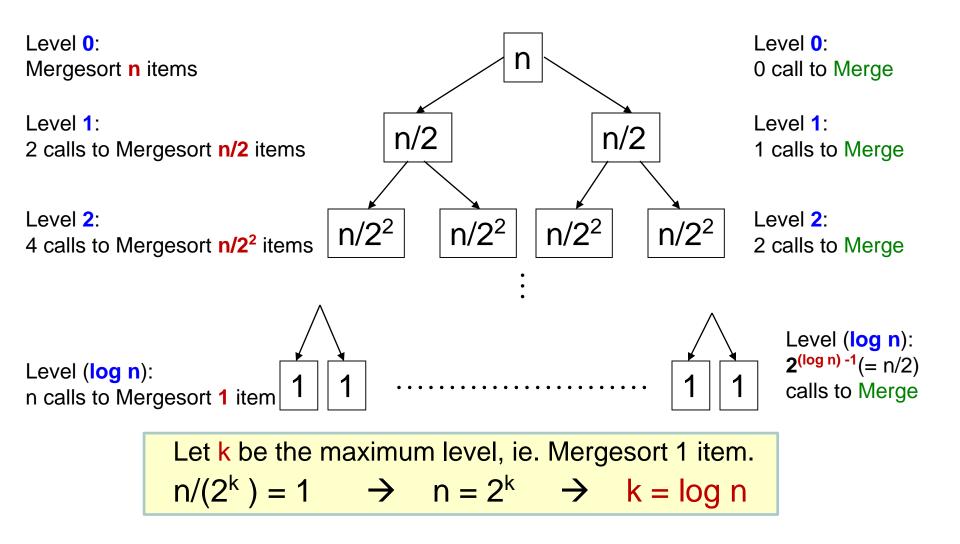
}

4 Analysis of Merge Sort (1/3)

- In Merge Sort, the bulk of work is done in the Merge step merge(a, i, mid, j)
- Total number of items = k = j i + 1
 - □ Number of comparisons $\leq k 1$ (Q: Why not = k 1?)
 - Number of moves from original array to temp array = k
 - Number of moves from temp array to original array = k
- In total, number of operations ≤ 3k − 1 = O(k)
- How many times is merge() called?

```
... mergeSort(int[] a, int i, int j) {
    if (i < j) {
        int mid = (i+j)/2;
        mergeSort(a, i, mid);
        mergeSort(a, mid+1, j);
        merge(a,i,mid,j);
    }
}</pre>
```

4 Analysis of Merge Sort (2/3)



4 Analysis of Merge Sort (3/3)

- Level 0: 0 call to Merge
- Level 1: 1 call to Merge with n/2 items each, $O(1 \times 2 \times n/2) = O(n)$ time
- Level 2: 2 calls to Merge with $n/2^2$ items each, $O(2 \times 2 \times n/2^2) = O(n)$ time
- Level 3: 2^2 calls to Merge with $n/2^3$ items each, $O(2^2 \times 2 \times n/2^3) = O(n)$ time
- Level (log n): 2^{(log n)-1}(= n/2) calls to Merge with n/2^{log n} (= 1) item each,

 $O(n/2 \times 2 \times 1) = O(n)$ time

In total, running time = (log n)*O(n) = O(n log n)

4 Drawbacks of Merge Sort

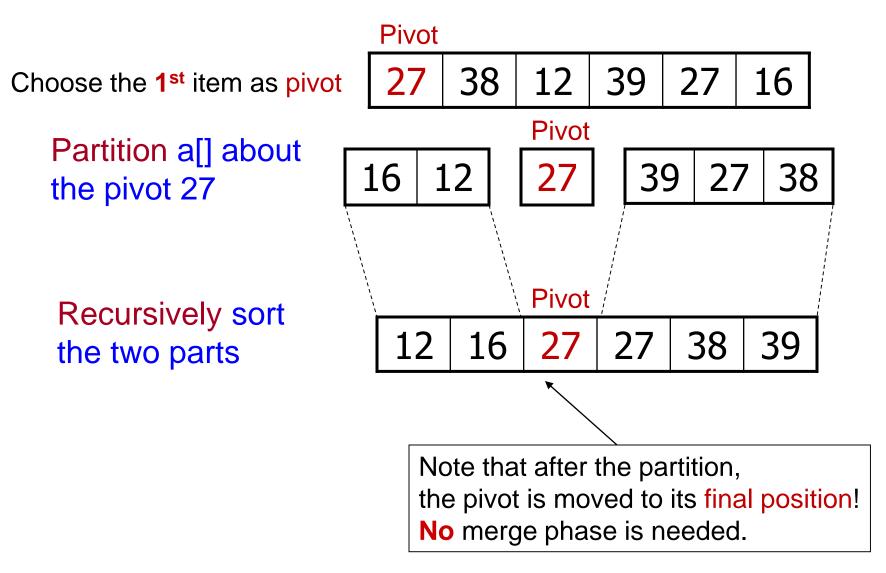
- Implementation of merge() is not straightforward
- Requires additional temporary arrays and to copy the merged sets stored in the temporary arrays to the original array
- Hence, additional space complexity = O(n)

Quick Sort

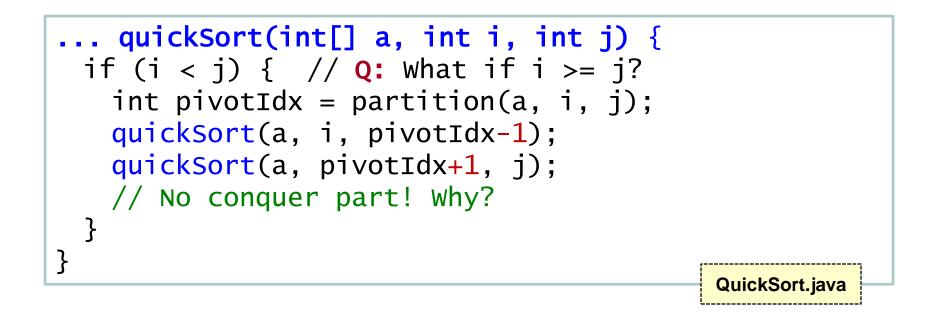
5 Idea of Quick Sort

- Quick Sort is a divide-and-conquer algorithm
- Divide Step: Choose a pivot item p and partition the items of a[i..j] into 2 parts so that
 Items in the first part are < p, and
 Items in the second part are ≥ p.
- Recursively sort the 2 parts
- Conquer Step: Do nothing! No merging is needed.
- What are the base cases?
- Note: Merge Sort spends most of the time in conquer step but very little time in divide step.
- **Q:** How about Quick Sort?
- Q: Is it similar to the Recursion lecture notes on finding the Kth smallest element?

5 Example of Quick Sort

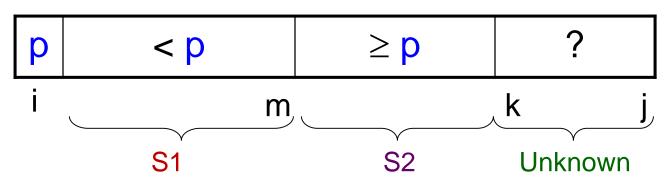


5 Code of Quick Sort



5 Partition algorithm idea (1/4)

- To partition a[i..j], we choose a[i] as the pivot p.
 Why choose a[i]? Are there other choices?
- The remaining items (i.e. a[i+1..j]) are divided into 3 regions:
 - \Box S1 = a[i+1..m] where items < p
 - □ S2 = a[m+1..k-1] where item \ge p
 - Unknown (unprocessed) = a[k..j], where items are yet to be assigned to S1 or S2.

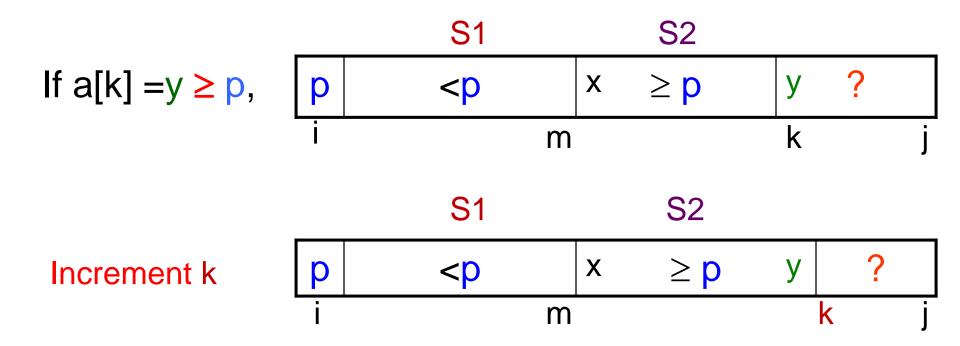


5 Partition algorithm idea (2/4)

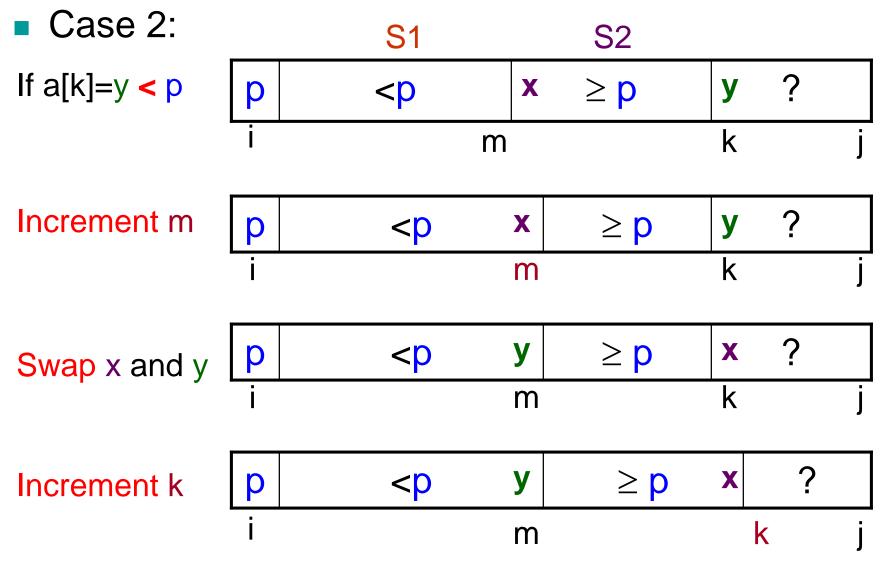
- Initially, regions S1 and S2 are empty. All items excluding p are in the unknown region.
- Then, for each item a[k] (for k=i+1 to j) in the unknown region, compare a[k] with p:
 - □ If $a[k] \ge p$, put a[k] into S2.
 - Otherwise, put a[k] into S1.
- Q: How about if we change ≥ to > in the condition part?

5 Partition algorithm idea (3/4)

Case 1:



5 Partition algorithm idea (4/4)

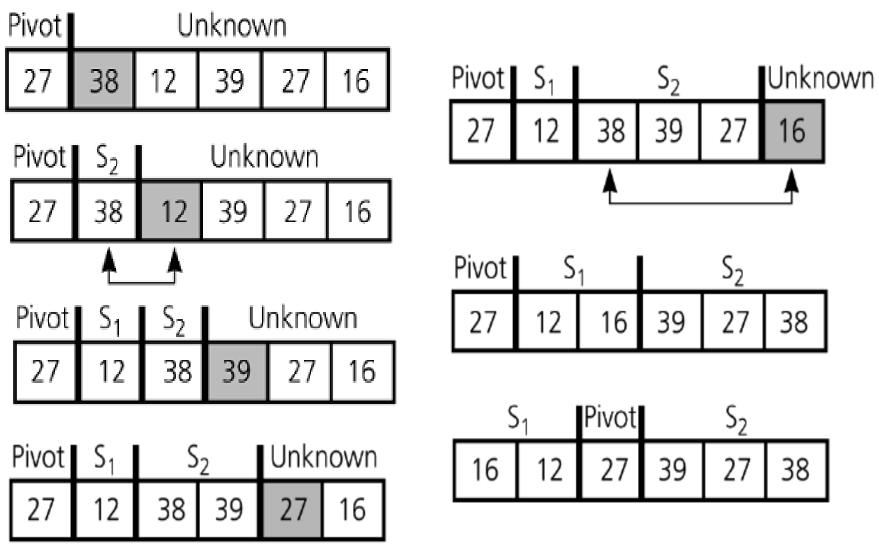


5 Code of Partition Algorithm

```
... partition(int[] a, int i, int j) {
 // partition data items in a[i..j]
 int p = a[i]; // p is the pivot, the i^{th} item
 int m = i; // Initially S1 and S2 are empty
 for (int k=i+1; k<=j; k++) { //process unknown region</pre>
   if (a[k] < p) \{ // case 2: put a[k] to S1
     m++;
     swap(a,k,m);
   } else { // case 1: put a[k] to S2. Do nothing!
   } // else part should be removed since it is empty
 }
 swap(a,i,m); // put the pivot at the right place
 return m; // m is the pivot's final position
}
```

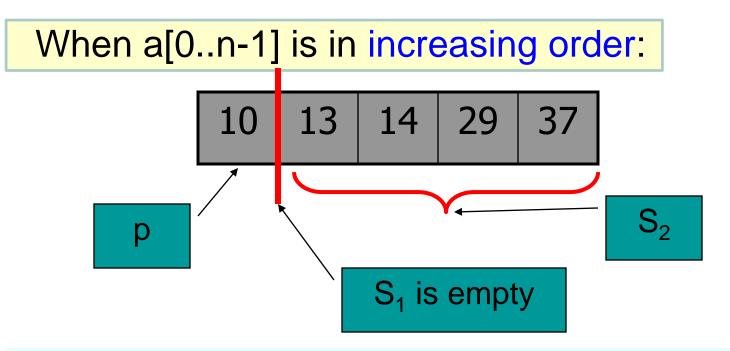
As there is only one 'for' loop and the size of the array is n = j - i + 1, so the complexity for partition() is O(n)

5 Partition Algorithm: Example



Same value, no need to swap them.

5 Analysis of Quick Sort: Worst Case (1/2)



What is the index returned by partition()?

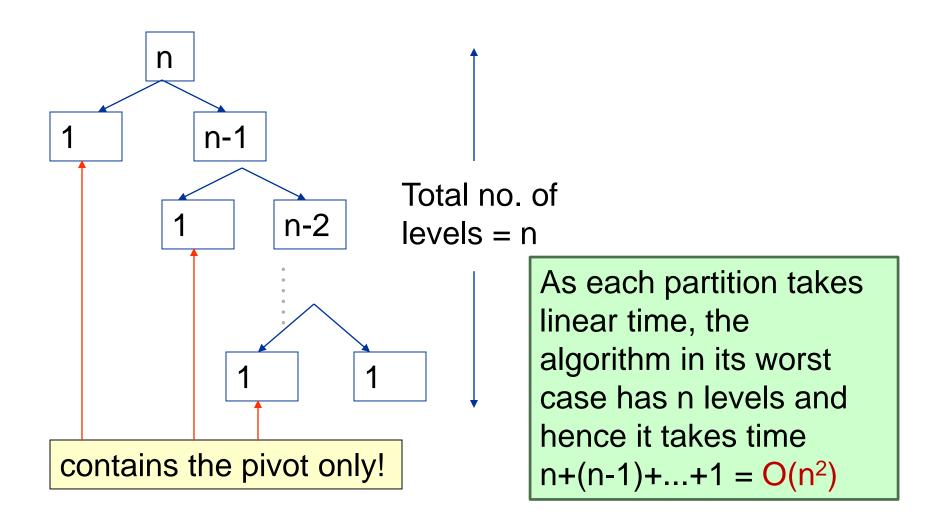
swap(a,i,m) will swap the pivot with itself!

The left partition (S1) is empty and

The right partition (S2) is the rest excluding the pivot.

What if the array is in decreasing order?

5 Analysis of Quick Sort: Worst Case (2/2)



5 Analysis of Quick Sort: Best/Average case

- Best case occurs when partition always splits the array into 2 equal halves
 - Depth of recursion is log n.
 - Each level takes n or fewer comparisons, so the time complexity is O(n log n)
- In practice, worst case is rare, and on the average, we get some good splits and some bad ones
- Average time is O(n log n)

6 Radix Sort

6 Idea of Radix Sort

- Treats each data to be sorted as a character string.
- It is not using comparison, i.e., no comparison among the data is needed.
- Hence it is a non-comparison based sort (the preceding sorting algorithms are called comparison based sorts)
- In each iteration, organize the data into groups according to the next character in each data.

6 Radix Sort of Eight Integers

0123, 2154, 0222, 0004, 0283, 1560, 1061, 2150 (156**0**, 215**0**) (106**1**) (022**2**) (012**3**, 028**3**) (215**4**, 000**4**) 1560, 2150, 1061, 0222, 0123, 0283, 2154, 0004 (00**0**4) (02**2**2, 01**2**3) (21**5**0, 21**5**4) (15**6**0, 10**6**1) (02**8**3) 0004, 0222, 0123, 2150, 2154, 1560, 1061, 0283 (0**0**04, 1**0**61) (0**1**23, 2**1**50, 2**1**54) (0**2**22, 0**2**83) (1**5**60) 0004, 1061, 0123, 2150, 2154, 0222, 0283, 1560 (**0**004, **0**123, **0**222, **0**283) (**1**061, **1**560) (**2**150, **2**154) 0004, 0123, 0222, 0283, 1061, 1560, 2150, 2154

Original integers Grouped by fourth digit Combined Grouped by third digit Combined Grouped by second digit Combined Grouped by first digit Combined (sorted)

6 Pseudocode and Analysis of Radix Sort

```
radixSort(int[] array, int n, int d) {
  // Sorts n d-digit numeric strings in the array.
   for (j = d down to 1) { // for digits in last position to 1<sup>st</sup> position
      initialize 10 groups (queues) to empty // Q: why 10 groups?
      for (i=0 through n-1) \{
          \mathbf{k} = \mathbf{j}^{\text{th}} \operatorname{digit} \operatorname{of} \operatorname{array}[\mathbf{i}]
          place array[i] at the end of group k
      Replace array with all items in group 0, followed by all items
      in group 1, and so on.
}
```

Complexity is $O(d \times n)$ where d is the maximum number of digits of the n numeric strings in the array. Since d is fixed or bounded, so the complexity is O(n).

7 Comparison of Sorting Algorithms

7 In-place Sort

- A sorting algorithm is said to be an in-place sort if it requires only a constant amount, i.e. O(1), of extra space during the sorting process.
- Merge Sort is <u>not</u> in-place. (Why?)
- How about Quick Sort and Radix Sort?

7 Stable Sort

- A sorting algorithm is stable if the relative order of elements with the same key value is preserved by the algorithm.
- Example 1 An application of stable sort:
 - Assume that names have been sorted in alphabetical order.
 - Now, if this list is sorted again by tutorial group number, a stable sort algorithm would ensure that all students in the same tutorial groups still appear in alphabetical order of their names.
- Quick Sort and Selection Sort are <u>not</u> stable. (Why?)

7 Non-Stable Sort

Example 2 – Quick Sort and Selection Sort are not stable:

Quick sort: 1285 5 150 4746 602 5 8356 // pivot in bold 1285 (5 150 602 5) (4746 8356) 5 5 150 602 1285 4746 8356 //pivot swapped with the last one in S1 // the 2 5's are in different order of the initial list

Selection sort: select the largest element and swap with the last one 1285 5 4746 602 5 (8356) 1285 5 5 602 (4746 8356) 602 5 5 (1285 4746 8356) 5 5 (602 1285 4746 8356) // the 2 5's are in different order of the initial list

7 Summary of Sorting Algorithms

	Worst Case	Best Case	In-place?	Stable?
Selection Sort	O(n ²)	O(n ²)	Yes	No
Insertion Sort	O(n ²)	O(n)	Yes	Yes
Bubble Sort	O(n ²)	O(n²)	Yes	Yes
Bubble Sort 2 (improved with flag)	O(n ²)	O(n)	Yes	Yes
Merge Sort	O(n log n)	O(n log n)	No	Yes
Radix Sort (non- comparison based)	O(n) (see notes 1)	O(n)	No	Yes
Quick Sort	O(n ²)	O(n log n)	Yes	No

Notes: 1. O(n) for Radix Sort is due to non-comparison based sorting.
2. O(n log n) is the <u>best possible</u> for comparison based sorting.

8 Use of Java Sort Methods

8 Java Sort Methods (in Arrays class)

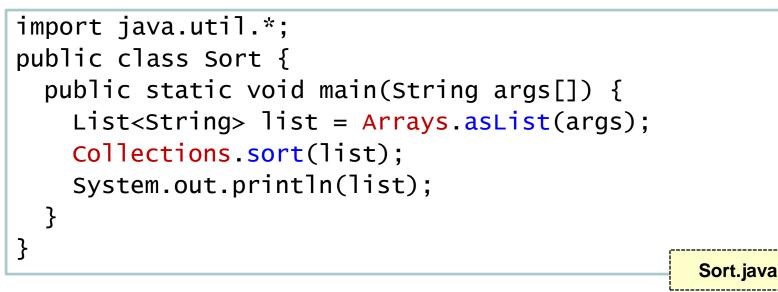
```
static void sort(byte[] a)
static void sort(byte[] a, int fromIndex, int toIndex)
static void sort(char[] a)
static void sort(char[] a, int fromIndex, int toIndex)
static void sort(double[] a)
static void sort(double[] a, int fromIndex, int toIndex)
static void sort(float[] a)
static void sort(float[] a, int fromIndex, int toIndex)
static void sort(int[] a)
static void sort(int[] a, int fromIndex, int toIndex)
static void sort(long[] a)
static void sort(long[] a, int fromIndex, int toIndex)
static void sort(Object[] a)
static void sort(Object[] a, int fromIndex, int toIndex)
static void sort(short[] a)
static void sort(short[] a, int fromIndex, int toIndex)
static <T> void sort(T[] a, Comparator<? super T> c)
static <T> void sort(T[] a, int fromIndex, int toIndex,
                      Comparator <? super T > c)
```

8 To use sort() in Arrays

- The entities to be sorted must be stored in an array first.
- If they are stored in a list, then we have to use Collections.sort()
- If the data to be sorted are not primitive, then Comparator must be defined and used

Note: Collections is a Java public class and Comparator is a public interface. Comparators can be passed to a sort method (such as Collections.sort()) to allow precise control over the sort order.

8 Simple program using Collections.sort()



Run the program:

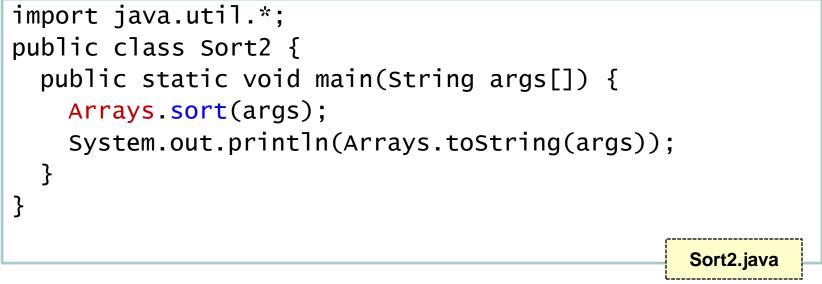
java Sort We walk the line

The following output is produced:

Note: Arrays is a Java public class and asList() is a method of Arrays which returns a fixed-size list backed by the specified array.

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8 Another solution using Arrays.sort()



Run the program:

java Sort2 We walk the line

The following output is produced:

8 Example: class Person

```
class Person {
 private String name;
 private int age;
 public Person(String name, int age) {
   this.name = name;
   this.age = age;
 public String getName() { return name; }
 public int getAge() { return age; }
 public String toString() {
   return name + " - " + age;
  }
}
                                           Person.java
```

8 Comparator: AgeComparator

```
import java.util.Comparator;
class AgeComparator implements Comparator<Person> {
  public int compare(Person p1, Person p2) {
    // Returns the difference:
    // if positive, age of p1 is greater than p2
   // if zero, the ages are equal
    // if negative, age of p1 is less than p2
    return p1.getAge() - p2.getAge();
  }
  public boolean equals(Object obj) {
    // Simply checks to see if we have the same object
    return this == obj;
  }
} // end AgeComparator
                                            AgeComparator.java
```

Note: compare() and equals() are two methods of the interface Comparator. Need to implement them.

8 Comparator: NameComparator

```
import java.util.Comparator;
class NameComparator implements Comparator<Person> {
 public int compare(Person p1, Person p2) {
   // Compares its two arguments for order by name
    return p1.getName().compareTo(p2.getName());
 public boolean equals(Object obj) {
   // Simply checks to see if we have the same object
   return this == obj;
} // end NameComparator
                                         NameComparator.java
```

8 TestComparator (1/3)

```
import java.util.*;
```

```
public class TestComparator {
  public static void main(String args[]) {
    NameComparator nameComp = new NameComparator();
    AgeComparator ageComp = new AgeComparator();
    Person[] p = new Person[5];
    p[0] = new Person("Michael", 15);
    p[1] = new Person("Mimi", 9);
    p[2] = new Person("Sarah", 12);
    p[3] = new Person("Andrew", 15);
    p[4] = new Person("Mark", 12);
    List<Person> list = Arrays.asList(p);
                                             TestComparator.java
```

8 TestComparator (2/3)

```
System.out.println("Sorting by age:");
    Collections.sort(list, ageComp);
    System.out.println(list + "\n");
    List<Person> list2 = Arrays.asList(p);
    System.out.println("Sorting by name:");
    Collections.sort(list2, nameComp);
    System.out.println(list2 + "\n");
    System.out.println("Now sort by age, then sort by name:");
    Collections.sort(list2, ageComp); // list2 is already
sorted by name
    System.out.println(list2);
  } // end main
} // end TestComparator
                                              TestComparator.java
```

8 TestComparator (3/3)

java TestComparator

Sorting by age: [Mimi - 9, Sarah - 12, Mark - 12, Michael - 15, Andrew - 15]

Sorting by name:

Now sort by age, then sort by name:

8 Another solution using Arrays.sort()

We can replace the statements

List<Person> list = Arrays.asList(p);
System.out.println("Sorting by age:");
Collections.sort(list, ageComp);
System.out.println(list + "\n");

with

System.out.println("Sorting by age using Arrays.sort():");
Arrays.sort(p, ageComp);
System.out.println(Arrays.toString(p) + "\n");

Summary

- We have introduced and analysed some classic sorting algorithms.
- Merge Sort and Quick Sort are in general faster than Selection Sort, Bubble Sort and Insertion Sort.
- The sorting algorithms discussed here are comparison based sorts, except for Radix Sort which is noncomparison based.
- O(n log n) is the best possible worst-case running time for comparison based sorting algorithms.
- There exist Java methods to perform sorting.

Links on Sorting Algorithms

- http://visualgo.net → http://visualgo.net/sorting.html
- <u>http://www.cs.ubc.ca/spider/harrison/Java/sorting-demo.html</u>
- <u>http://max.cs.kzoo.edu/~abrady/java/sorting/</u>
- <u>http://www.sorting-algorithms.com/</u>
- <u>http://en.wikipedia.org/wiki/Sort_algorithm</u>
- http://search.msn.com/results.aspx?q=sort+algorithm&FORM =SMCRT
- and others (please google)

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