

## CHAPTER 2

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# PROBLEM SOLVING

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### **In this chapter you will learn about**

- ❖ The craft of problem solving, and the mental activities it entails.
- ❖ The need for creativity in problem solving, and some fallacies of creative thinking.

## 2.1 Preliminaries

**Problem-1:** You are back from a leisure stroll in the park, and discover that you have dropped your wallet that is of great sentimental value to you. How do you retrieve your lost wallet?

In our daily life, we encounter problems big and small. Some are easy to solve; some are tough nuts to crack. Some call for a structured solution; some require an unstructured approach. Some are interesting; some are not.

As for the nature of problems, they range from the mathematical to the philosophical. In computing, we deal with *algorithmic problems* whose solutions are expressed as algorithms. An algorithm is a set of well-defined steps and instructions that can be translated into a form, usually known as the code or program, that is executable by a computing device.

You will learn more about algorithmic problem solving in Chapter 6. For the time being, we shall study problem solving in general. However, we still want to confine our domain within the logical applications of scientific and mathematical methods. Therefore, we will exclude problems such as “How could President Clinton survive the Lewinsky scandal?”, “How to become a millionaire in 21 days?”, “How to score straight A’s in examinations?”, and “How many computer scientists does it take to change the light bulb?”

What a big disappointment!

Returning to the wallet-in-the-park problem, have you found a solution?

If you have, then something is fishy. The problem statement is incomplete. Where did you actually drop the wallet? Did you assume that it was dropped at the park? Could it have happened on the way home? If the wallet was indeed dropped at the park, what is the park like?

So, that was a trick question?

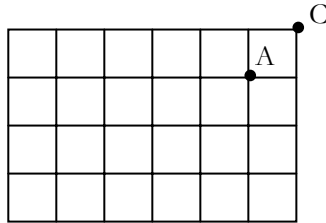
These questions (and more, if you can think of) must be answered before you could reach a complete understanding of the problem. Assumptions should not be made without basis. Irrelevant information (such as the sentimental value of the wallet) should not get in the way. Incomplete information must be sought. You could only fully understand the problem after all ambiguities are removed. Only then should you set out to solve the problem. Otherwise, without a proper term of reference, you might end up in an interesting situation described in Article 2-1.

Now, supposed your wallet was lost in the rectangular park 2km by 1km. How do you recover the wallet?

Your wallet is lying  
somewhere in this  
region.



**Problem-5:** Given a rectangular grid as shown below. You may move from one intersection to another, governed by the rule that you can travel only upwards or to the right, along the grid lines. How many paths are there from each of the intersections to the top-right corner of the grid? For example, there are two paths from point A to point C, one that goes right and then turns upwards, and the other that goes up and turns right.



How is this problem related to the Pascal's triangle? Knowing the relation, how would you use Pascal's triangle to solve this problem?

This illustrates the oft-encountered situation that calls for relating a problem on hand to one that we have solved before. By consciously seeking out the similarity among problems, and performing problem transformation, these help us to sharpen our problem solving dexterity.

## 2.2 The Problem Solving Process

*A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on mind and character for a lifetime.*

– George Pólya

The problems in the preceding section offer a glimpse into the craft of problem solving. The Hungarian mathematician George Pólya (1887-1985) conceptualised the process and captured it in the famous book “How To Solve It” [1]. Though the treatment is mathematical in nature, the book offers valuable insights that are well applicable to any scientific field.

In his book, Pólya enumerates four phases a problem solver has to go through. First, we need to understand the problem, and clarify any doubts about the problem statement if necessary, as we have discussed in Problem-1. Second, we need to find the connection between the data and the unknown, to make out a plan for the solution. Auxiliary problems might be created in the process. Third, we are to carry out our plan, checking the validity of each step. Finally, we have to review the solution.

There are many  
questions which fools  
can ask that wise  
men cannot answer.

- G. Pólya

The four phases are outlined below. Note that asking questions is a running theme in all phases of the process. In fact, asking questions is part and parcel of learning (refer to the wheel of learning). As such, you should start cultivating an inquisitive mind today, if you have not done so.

❖ ***Phase 1: Understanding the problem.***

- ♦ *What is the unknown? What are the data?*
- ♦ *What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?*
- ♦ *Draw a figure. Introduce suitable notation.*
- ♦ *Separate the various parts of the condition. Can you write them down?*

❖ ***Phase 2: Devising a plan.***

- ♦ *Have you seen it before? Or have you seen the same problem in slightly different form?*
- ♦ *Do you know a related problem?*
- ♦ *Look at the unknown! Try to think of a familiar problem having the same or similar unknown.*
- ♦ *Split the problem into smaller, simpler sub-problems.*
- ♦ *If you cannot solve the proposed problem try to solve first some related problem. Or solve a more general problem. Or a special case of the problem. Or solve a part of the problem.*

❖ ***Phase 3: Carrying out the plan.***

- ♦ *Carrying out your plan of the solution, check each step.*
- ♦ *Can you see clearly that the step is correct?*
- ♦ *Can you prove that it is correct?*

❖ ***Phase 4: Looking back.***

- ♦ *Can you check the result?*
- ♦ *Can you derive the result differently?*
- ♦ *Can you use the result, or the method, for some other problem?*

## 2.3 Creative Thinking

*Most new discoveries are suddenly-seen things that were always there. A new idea is a light that illuminates presences while simply had no form for us before the light fell on them.*

– Susan Langer

The dissection on the problem solving process produces a finding that guides, not prescribes. It provides questions, not answers. It is a checklist, not a panacean device. Hence, the opportunities of solving problems do not cease with the revelation. In fact, this is just the beginning.

A rich knowledge base and effective techniques are requisites for problem solving. However, there appears to be another quality that makes some better problem solvers than others, though they may be equally knowledgeable. They call it creativity.

Many are intrigued by the ingenuity and creativity of prominent personalities the like of Thomas Edison, Van Gogh, and Albert Einstein, and hope to acquire this asset to apply it in problem solving. Creative thinking has become the buzzword these days. What exactly is creativity? Can it be taught? In Chapter 1, we looked at what learning is not. Here, we shall adopt the same approach and dispel some myths around creative thinking. The following pointers are found in the book “Creative Thinking & Problem Solving” [2].

“Creativity is seeing what’s really there.”

– Sampson Raphaelson

### ❖ *Myth 1: Creativity requires genius.*

Get yourself or your mental straitjacket out of the way; believe in yourself and in the possibility of thinking more intuitively and creatively.

### ❖ *Myth 2: You have to be odd to really turn on the imagination afterburners.*

Creative thinking is sound, not strange, reasoning. To create, you must be mentally and emotionally flexible yet persistent enough to extract and recombine threads from daily experience, your knowledge and image bank, the senses, and intuition.

### ❖ *Myth 3: Creative thinking isn’t rigorous, but uses simple processes.*

Real creative thinking takes practice and involves carefully following principles and effective techniques.

The ultimate creative guy produces excellent work even when his hands are tied.

So you see, there is no big secret behind creative thinking. Creativity does not spurt out of a vacuum, but is the result of a concoction of knowing the principles, applying the right methods, plus sheer concentration and purposeful determination.

Another misperceived notion about creativity is that all constraints must be lifted before creativity can take reign. Often we heard students complain about restrictions imposed on their assignments, for example, the barring of certain methods in their solution. They charged that such a ban amounted to stifling their creativity. What they did not realise is that on the contrary, the best opportunities for creativity are such exacting times when the resources are limited or constrained. Creativity is not equivalent or tied to freedom.

Neither is creativity characterised by an absence of direction. When creative thinking is applied to problem solving, the mind wanders in search for ideas, of which some may be the wildest or the most unorthodox, but the objective has to stay in the back of the solver's mind. This reiterates the importance of understanding the problem, and it brings out the issue on problem definition.

The authors of the book “Strategies for Creative Problem Solving” [3] have this to say about the importance of producing correct problem definitions in handling real-life situations.

*“Problem definition is a common but difficult task because true problems are often disguised in a variety of ways. It takes a skilful individual to analyse a situation and extract the real problem from a sea of information. Ill-defined or poorly posed problems can lead novice (and not so novice) engineers down the wrong path to a series of impossible or spurious solutions. Defining the ‘real problem’ is critical to finding a workable solution.”*

– H. Scott Fogler and Steven E. LeBlanc

Ill-defined problems may mislead the solver into diverting his attention to the wrong problems, as illustrated in a real-life case in Article 2-2.

## 2.4 Summary

This chapter presents the four phases of problem solving: understanding the problem, devising a plan, executing the plan, and reviewing the solution. Some examples are given to illustrate some of the mental operations involved.

Creative thinking is discussed in the context of problem solving. A few myths on creativity are exposed, and some common misconceptions and associated pitfalls are identified.

This chapter serves as a prelude to Chapter 3 on Algorithmic Problem Solving.

**Article 2-1: Barometer**

SOME TIME ago I received a call from a colleague, who asked if I would be the referee on the grading of an examination question. He was about to give a student a zero for his answer to a physics question, while the student claimed he should receive a perfect score and would if the system were not set up against the student. The instructor and the student agreed to an impartial arbiter, and I was selected. I went to my colleague's office and read the examination question: "Show how it is possible to determine the height of a tall building with the aid of a barometer."

The student had answered: "Take the barometer to the top of the building, attach a long rope to it, lower it to the street, and then bring it up, measuring the length of the rope. The length of the rope is the height of the building."

I pointed out that the student really had a strong case for full credit since he had really answered the question completely and correctly. On the other hand, if full credit were given, it could well contribute to a high grade in his physics course. A high grade is supposed to certify competence in physics, but the answer did not confirm this. I suggested that the student have another try at answering the question. I was not surprised that my colleague agreed, but I was surprised when the student did.

I gave the student six minutes to answer the question with the warning that the answer should show some knowledge of physics. At the end of five minutes, he had not written anything. I asked if he wished to give up, but he said no. He had many answers to this problem; he was just thinking of the best one. I excused myself for interrupting him and asked him to please go on. In the next minute, he dashed off his answer which read:

"Take the barometer to the top of the building and lean over the edge of the roof. Drop the barometer, timing its fall with a stopwatch. Then, using the formula  $x=0.5 \cdot a \cdot t^2$ , calculate the height of the building."

At this point, I asked my colleague if he would give up. He conceded, and gave the student almost full credit. In leaving my colleague's office, I recalled that the student had said that he had other answers to the problem, so I asked him what they were.

"Well," said the student, "there are many ways of getting the height of a tall building with the aid of a barometer. For example, you could take the barometer out on a sunny day and measure the height of the barometer, the length of its shadow, and the length of the shadow of the building, and by the use of simple proportion, determine the height of the building."

"Fine," I said, "and others?"



"Yes," said the student. "There is a very basic measurement method you will like. In this method, you take the barometer and begin to walk up the stairs. As you climb the stairs, you mark off the length of the barometer along the wall. You then count the number of marks, and this will give you the height of the building in barometer units.

"A very direct method."

"Of course. If you want a more sophisticated method, you can tie the barometer to the end of a string, swing it as a pendulum, and determine the value of  $g$  at the street level and at the top of the building. From the difference between the two values of  $g$ , the height of the building, in principle, can be calculated."

"On this same tact, you could take the barometer to the top of the building, attach a long rope to it, lower it to just above the street, and then swing it as a pendulum. You could then calculate the height of the building by the period of the precession".

"Finally," he concluded, "there are many other ways of solving the problem. Probably the best," he said, "is to take the barometer to the basement and knock on the superintendent's door. When the superintendent answers, you speak to him as follows: 'Mr. Superintendent, here is a fine barometer. If you will tell me the height of the building, I will give you this barometer.'"

At this point, I asked the student if he really did not know the conventional answer to this question. He admitted that he did, but said that he was fed up with high school and college instructors trying to teach him how to think.

- Author unknown.

### **Article 2-2: Impatient Guests**

SHORTLY after the upper floors of a high-rise hotel had been renovated to increase the hotel's room capacity, the guests complained that the elevators were too slow. The building manager assembled his assistants. His instructions to solve the perceived problem: "Find a way to speed up the elevators." After calling the elevator company and an independent expert on elevators, it was determined that nothing could be done to speed up the elevators.

Next, the manager's direction was "Find a location and design a shaft to install another elevator." An architectural firm was hired to carry out this request. However, neither the shaft nor the new elevator was installed because shortly after the firm was hired the real problem was uncovered. The real problem was to find a way to take the guests' minds off their wait rather than to install more elevators. The guests stopped complaining when mirrors were installed on each floor in front of the elevators

*Adapted from Chemtech, 22, 1, pp 24, 1992.*

## Exercises

The questions below are of varied nature. The selection is chosen to illustrate the extent of general problem solving. Different kinds of questions require different domain knowledge and strategies. Apply your problem solving skill and creativity here!

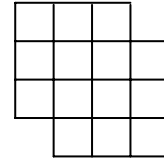
1. A bear, starting from the point  $P$ , walked one mile due south. Then he changed direction and walked one mile due east. Then he turned again to the left and walked one mile due north, and arrived at the point  $P$  he started from. What was the colour of the bear?
2. Two towns  $A$  and  $B$  are 3 kilometres apart. It is proposed to build a new school serving 100 students in town  $A$  and 50 students in town  $B$ . How far from town  $A$  should the school be built if the **total distance travelled by all 150 students is to be as small as possible**?
3. A traveller arrives at an inn. He has no money but only a silver chain consisting of 6 links. He uses one link to pay for each day spent at the inn, but the innkeeper agrees to accept **no more than one broken link**. How should the traveller cut up the chain in order to settle accounts with the innkeeper on a daily basis?
4. What is the **least number of links** that have to be cut if the traveller stays 100 days at the inn and has a chain consisting of 100 links? What is the answer in the general case ( $n$  days and  $n$  links)?
5. The minute and hour hands of a clock coincide exactly at 12 o'clock. At what time later do they first **coincide again**?
6. Six glasses are in a row, the first three full of juice, the second three empty. By moving **only one glass**, can you arrange them so that empty and full glasses alternate?
7. You throw away the outside and cook the inside. Then you eat the outside and throw away the inside. What did you eat?
8. Rearrange the letters in the words **new door** to make one word.
9. A mad scientist wishes to make a chain out of plutonium and lead pieces. There is a problem, however. If the scientist places two pieces of plutonium next to each other, **BOOM!** The question is, in how many ways can the scientist safely construct a chain of length  $n$ ?



10. Among 12 ball bearings, one is defective, but it is not known if it is heavier or lighter than the rest. Using a traditional balance (with two pans hanging down the opposite ends of a lever supported in the middle), how do you determine which is the defective ball bearing, and whether it is heavier or lighter than the others, within three attempts?
11. Figure 1 below shows a domino and Figure 2 shows a  $4 \times 4$  board with the two squares at opposite corners removed. How do you show that it is not possible to cover this board completely with dominoes? All dominoes should lay flat on the board, without overlapping one another, and should not cover area outside the board.



Figure 1. A domino.

Figure 2. A  $4 \times 4$  board with 2 corner squares removed.

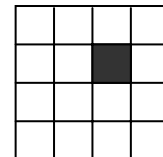
*General case:* How do you show the same for an  $n \times n$  board with the two squares at opposite corners removed, where  $n$  is even?

*Special case:* How do you show the same for an  $n \times n$  board with the two squares at opposite corners removed, where  $n$  is odd?

12. This problem requires the Principle of Mathematical Induction. Figure 3 below shows a triomino and Figure 4 shows a  $4 \times 4$  board with a defect (hole) in one square.



Figure 3. A triomino.

Figure 4. A  $4 \times 4$  board with a hole.

How do you show that the board can be covered with triominos?

*General case:* How do you show that a  $2^n \times 2^n$  board (where  $n \geq 0$ ) with a hole in one square (anywhere on the board) can be covered with triominos?

## References:

- [1] Pólya, George, *How To Solve It: A New Aspect of Mathematical Method*, Princeton University Press, 2<sup>nd</sup> edition, 1988.
- [2] Fabian, John P., *Creative Thinking & Problem Solving*, Lewis Publishers, 1990.
- [3] H. Scott Fogler, and Steven E. LeBlanc, *Strategies for Creative Problem-Solving*, Prentice Hall, 1995.