CS1231S Assignment #1

AY2025/26 Semester 1

Deadline: Monday, 15 September 2025, 1:00pm

IMPORTANT: Please read the instructions below carefully.

This is a graded assignment worth 10% of your final grade. There are **six questions** (an admin Q0 and six task questions Q1–6) with a total score of 40 marks. Please work on it by yourself, not in a group or discussion/in collaboration with anybody. Anyone found committing plagiarism (submitting other's work as your own), or sending your answers to others, or other forms of academic dishonesty, will be penalised with a straight zero for the assignment, and reported to the school. Please see SoC website "Preventing Plagiarism" https://www.comp.nus.edu.sg/cug/plagiarism/.

You must submit your assignment to **Canvas > Assignments > Assignment 1 submission** before the deadline.

Your answers may be typed or handwritten. Make sure that it is legible (for example, don't use very light pencil or ink if it is handwritten, or font size smaller than 11 if it is typed) or marks may be deducted.

You are to submit a SINGLE pdf file, where each page is A4 size. Do not submit files in other formats. If you submit multiple files, only the last submitted file will be graded.

Late submission will NOT be accepted. We have set the closing time of submission to slightly after 1pm to give you a few minutes of grace, but in your mind, you should treat **1pm** as the deadline. If you think you might be too busy on the day of the deadline, please submit earlier. Also, avoid submitting in the last few minutes, as the system may get sluggish due to overload and you will miss the deadline, and we will not extend the deadline for you.

Note the following:

- Name your pdf file with your Student Number. Your student number begins with 'A' (eg: A0234567X). (Do not mix up your student number with your NUSNET-id which begins with 'e'.)
- At the top of the first page of your submission, write your Name and Tutorial Group.
- To keep the submitted file short, please submit your answers without including the questions.
- As this is an assignment given well ahead of time, we expect you to work on it early. You should submit polished work, not answers that are untidy or appear to have been done in a hurry, for example, with scribbling and cancellation all over the places. Marks may be deducted for untidy work.
- It is always good to be clear and leave no gap so that the marker does not need to make guesswork on your answer. When in doubt, the marker would usually not award the mark.
- Do not use any methods that have not been covered in class. When using a theorem or result that has appeared in class (lectures or tutorials), please quote the theorem number/name, the lecture and slide number, or the tutorial number and question number in that tutorial, failing which marks may be deducted. Remember to use numbering and give justification for important steps in your proof, or marks may be deducted.

To combine all pages into a single pdf document for submission, you may find the following scanning apps helpful if you intend to scan your handwritten answers:

for Android: https://fossbytes.com/best-android-scanner-apps/

for iphone: https://www.switchingtomac.com/tutorials/ios-tutorials/the-best-ios-scanner-apps-to-scan-documents-images/

If you need any clarification about this assignment, please do NOT email us or post on telegram, but post on the Canvas > Discussions> Assignments forum or on QnA "Assignment" topic so that everybody can read the answers to the queries.

Question 0. (Total: 2 marks)
Check that …

- you have submitted a pdf file with your Student Number as the filename. [1 mark]
- you have written <u>both</u> your name and tutorial group number (eg: T02) at the top of the first page of your file. (If you miss either one you will not be given any mark.) [1 mark]

Question 1. Propositional logic (Total: 8 marks)

(a) Draw a truth table to prove each of the following statements. For truth tables, you may use **T** for true and **F** for false.

(i) $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$ [2 marks]

(ii) $(q \vee p) \wedge (r \vee p) \wedge (r \vee \sim q) \equiv (q \vee p) \wedge (r \vee \sim q)$ [2 marks]

Answer:

p	q	r	$q \wedge r$	$p \rightarrow q$	$p \rightarrow r$	L_i	R_i
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F
T	F	T	F	F	T	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

p	q	r	$q \vee p$	$r \vee p$	$r \vee \sim q$	L_{ii}	R_{ii}
T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	T	T	F	F
F	F	F	F	F	T	F	F

Marking scheme:

LHS for each statement 1 mark each. RHS for each statement 1 mark each.

(b) Simplify the statement form below in <u>no more than 14 steps</u>. Make sure you do not skip any step, and every step must be justified by a law. You may use a single law multiple times in a single statement and cite it once. For instance, if you use commutative law two times in a single step you may cite it as: (commutative law * 2). Use **true** and **false** for tautology and contradiction respectively. (This question checks that you apply the laws rigorously and cite them correctly, so we will be strict in our grading.) [4 marks]

$$\sim((p \to q) \wedge (q \to r))$$

The simplified answer must be in the following format.

$$(\cdot \vee \cdot) \wedge (\cdot \vee \cdot)$$

Answer:

$\sim\big((p \to q) \wedge (q \to r)\big)$	
$\equiv \sim(p \to q) \vee \sim(q \to r)$	(De Morgan's law)
$\equiv \sim(\sim p \vee q) \vee \sim(\sim q \vee r)$	(Implication law * 2)
$\equiv (\sim(\sim p) \wedge \sim q) \vee (\sim(\sim q) \wedge \sim r)$	(De Morgan's law * 2)
$\equiv (p \wedge \sim q) \vee (q \wedge \sim r)$	(Double Negation * 2)
$\equiv \big((p \wedge \sim q) \vee q\big) \wedge \big((p \wedge \sim q) \vee \sim r\big)$	(Distributive law)
$\equiv \big(q \vee (p \wedge \sim q)\big) \wedge \big(\sim r \vee (p \wedge \sim q)\big)$	(Commutative law * 2)
$\equiv \big((q \vee p) \wedge (q \vee \sim q)\big) \wedge ((\sim r \vee p) \wedge (\sim r \vee \sim q))$	(Distributive law * 2)
$\equiv \big((q \vee p) \wedge true\big) \wedge ((\sim r \vee p) \wedge (\sim r \vee \sim q))$	(Universal Bound Law)
$\equiv ((q \vee p)) \wedge (\sim r \vee p) \wedge (\sim r \vee \sim q)$	(Identity Law)
$\equiv (q \vee p) \wedge (\sim r \vee p) \wedge (\sim r \vee \sim q)$	(Associative Law)
$\equiv (q \vee p) \wedge (\sim r \vee \sim q)$	(Using Q1a(ii))

Marking scheme:

0.5 marks are deducted for every incorrect use of the law.

2 marks are deducted if the final answer is not in the specified simplified form.

Question 2. Argument (Total: 5 marks)
Determine whether the following argument is valid or invalid. Let $P1, P2$ and $P3$ denote the premises with p, q, and r as the statement variables:

$$P1 \equiv p \to \sim q$$
$$P2 \equiv \sim r \to p$$
$$P3 \equiv q$$
$$\therefore r \vee p$$

You should <u>begin your answer by stating "**Valid**" or "**Invalid**"</u>, followed by your proof. You may refer to premises as P1, P2, P3 instead of their statement form. You are not allowed to show your truth table (partial or full) in your answer, though you may use truth table in your own rough work to help you derive your proof.

Answer. Valid (1 mark).

For P2 and P1, by transitive rule of inference $\sim r \to \sim q$, call it P4.

From P4 and P3, by modus tollens r. Students may do contrapositive followed by modus ponens.

By generalisation we obtain the desired consequence.

Marking scheme for proof (out of 4 marks):

If students use any truth table or critical rows or proof involves trying different truth values, no marks should be awarded.

If the solution uses laws of inference and they forget to cite transitive rule, there is no need to penalise the proof.

If the solution uses logical equivalence (most common showing P2 is equivalent to Conclusion) and does not justify Specialisation, 1 mark is deducted. 1 mark is deducted for every unjustified step in deriving logical equivalence (such as missing implication law, contrapositive, etc). Thus, a well justified proof missing specialisation will receive 3 marks.

Question 3. Sets (Total: 5 marks)
No working/justification is required for this question.

For $k \in \mathbb{N}$ the set A_k is defined as $A_k = \{n \in \mathbb{N} \mid 0 \leq n \leq 5k\}$ and the set B_k is defined as $B_k = \{n + k \mid n \in \mathbb{N}\}$. Write down answers for the following questions.

a. $A_2 \cap B_2 = X \cup Y$ where X and Y are mutually disjoint sets. Write down sets X and Y in the set- roster notation. [1 mark]
b. Fill in the blank to make the following statement true: $\forall k \in \mathbb{N}\ A_k \cup B_k = B$ [1 mark]
c. What is the cardinality of $A_k \cap A_{k+1}$? [1 mark]
d. What is the cardinality of $B_k \setminus B_m$ for any two natural numbers k and m? [2 marks]

Answer.

a. One possible solution is: $X = \{2, 3, 4, 5\},\ \ Y = \{6, 7, 8, 9, 10\}$.
b. 0.
c. $5k + 1$
d. If $m \geq k, m - k$. Otherwise, it is zero. (one mark is deducted if m=k is not handled)

Question 4. Predicate Logic (Total: 8 marks)
Let S denote the set of all students and T denote the set of all tutorial classes. Define the predicate $R(x, y)$ to mean "student $x \in S$ is registered for tutorial class $y \in T$". Express each of the following statements in formal languages using predicates and quantifiers.

(a) Every student is registered in at most one tutorial. [2 marks]
(b) Negation of the statement (a). [2 marks]
(c) There is a tutorial class with exactly one student registered in it. [2 marks]
(d) Negation of the statement (c). [2 marks]

Answer. This is the hardest question in the assignment. (a) and (c) are tough but negations should be mechanical applications of the rules.

a. $\forall\, s \in S\ \forall\, t_1, t_2 \in T\ \ (R(s, t_1) \wedge R(s, t_2) \rightarrow t_1 = t_2)$.
b. $\exists\, s \in S\ \exists\, t_1, t_2 \in T\ \ R(s, t_1) \wedge R(s, t_2) \wedge (t_1 \neq t_2)$.

c. $\exists\, t \in T\; \exists\, s \in S\; \big(R(s,t) \wedge (\forall\, s' \in S\; R(s',t) \rightarrow s = s')\big)$.

Alternative solution: $\exists y \in T\; (\exists x \in S\; R(s,t) \wedge \forall x_1, x_2 \in S\; (R(x_1, y) \wedge R(x_2, y)) \rightarrow x_1 = x_2)$

d. $\forall\, t \in T\; \forall s \in S\; ({\sim}R(s,t) \vee \exists\, s' \in S\; (R(s',t) \wedge (s \neq s')))$.

If domains are missing in all statements 2 marks are deducted(1 for each missing domain) – assuming the rest of the statements are correct.

For any non-trivial (a predicate form that involves logical connectives and at least two predicate variables) incorrect solutions of part (a) and part(c), if an accurate negation of the (incorrect) solutions, 1 mark is awarded each for (b) and (d).

Question 5. Proof (6 marks)

Complete the following proof that proves the following statement:

$$\forall a, b, c \in \mathbb{Z}\;\; (a^2 + b^2 = c^2) \rightarrow (Even(a) \vee Even(b))$$

Predicate $Even(a)$ evaluates to true if a is an even number, otherwise it evaluates to false.

Proof:

1. Let $a, b\ and\ c$ be any three integers.
2. Assume both $a\ and\ b$ are odd integers such that $a^2 + b^2 = c^2$.
3. ….

Answer. This is an interesting question wherein providing a direct proof as well as a proof by contraposition is very tricky (how would one introduce c?). I wanted to directly ask the question, but I indirectly gave the hint of the proof by contradiction by providing Line 1 and Line 2.

4. Let $a, b\ and\ c$ be any three integers.
5. Assume that a and b are odd and $a^2 + b^2 = c^2$.
   - 2.1 Using the definition of odd numbers, there exists two integers x and y such that $a = 2x + 1$ and that $b = 2y + 1$.
   - 2.2 Consider,
   $$\begin{aligned} a^2 + b^2 &= (2x+1)^2 + (2y+1)^2 \\ &= 4(x^2 + y^2 + x + y) + 2 \quad (by\ simple\ algebra) \end{aligned}$$
   - 2.3 Thus, $a^2 + b^2 \equiv 2\ (mod\ 4)$.
   - 2.4 Case I: c is even.
     - 2.4.1 Using the definition of even number, there exists an integer x such that $c = 2x$.
     - 2.4.2 Consider $c^2 = 4\,x^2$.
     - 2.4.3 Thus, $c^2 \equiv 0\ (mod\ 4)$.
     - 2.4.4 Contradiction to Line 2.3
   - 2.5 Case II: c is odd.
     - 2.5.1 Using the definition of odd number, there exists an integer x such that $c = 2x + 1$.
     - 2.5.2 Consider $c^2 = 4\,(x^2 + x) + 1$.
     - 2.5.3 Thus, $c^2 \equiv 1\ (mod\ 4)$.
     - 2.5.4 Contradiction to Line 2.3
   - 2.6 In either case, we get a contradiction.
6. Therefore, if $a^2 + b^2 = c^2$ then either a or b is even

Marking scheme: 2 marks for arriving at Line 2.3. 2 marks for each case. For every case, 1 mark is awarded if the contradiction is appropriately established.

Alternatively, after Line 2.3 one can directly say c is even. Justification should be: $odd(a) \leftrightarrow odd(a^2), odd(a) \land odd(b) \rightarrow even(a+b)$ or Proposition 4.6.4. If the justification is missing 2 marks are deducted.

Question 6. Proof on Sets (6 marks)
Prove for any sets A and B: $A \subseteq B \leftrightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B)$.

Answer.

Proof for the forward direction $A \subseteq B \rightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B)$

1. Assume $A \subseteq B$.
2. Let X be any set such that $X \in \mathcal{P}(A)$.
   2.1 By the definition of the power set, $X \subseteq A$.
   2.2 By transitivity of the subset (cite) $X \subseteq B$.
   2.3 By the definition of the power set, $X \in \mathcal{P}(B)$.
3. Therefore, by the definition the subset $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

Proof for the backward direction $A \subseteq B \leftarrow \mathcal{P}(A) \subseteq \mathcal{P}(B)$

1. Assume $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.
2. If A is an empty set, $A \subseteq B$.
3. If A is not an empty set, let x be any element such that $x \in A$.
   2.1 By the definition of the power set, $\{x\} \in \mathcal{P}(A)$.
   2.2 By the definition of subset, $\{x\} \in \mathcal{P}(B)$
   2.3 By the definition of the power set, $x \in B$.
4. Therefore, by the definition the subset $A \subseteq B$.

Marking scheme: This proof is meant to assess student's ability to stick to the structure. Deduct 1 mark if the proof provided long worded arguments in English.

3 marks forward direction (1 for power set, 1 for transitivity, 1 for overall correctness in the proof) and 3 marks for backward direction.

Each of the directions can be proves in other ways such as contraposition or contradiction. Any missing justifications will be penalised.

=== End of paper ===