CS1231S Assignment #1

AY2025/26 Semester 1

Deadline: Monday, 15 September 2025, 1:00pm

IMPORTANT: Please read the instructions below carefully.

This is a graded assignment worth 10% of your final grade. There are **six questions** (an admin Q0 and six task questions Q1–6) with a total score of 40 marks. Please work on it by yourself, not in a group or discussion/in collaboration with anybody. Anyone found committing plagiarism (submitting other's work as your own), or sending your answers to others, or other forms of academic dishonesty, will be penalised with a straight zero for the assignment, and reported to the school. Please see SoC website "Preventing Plagiarism" https://www.comp.nus.edu.sg/cug/plagiarism/.

You must submit your assignment to **Canvas > Assignments > Assignment 1 submission** before the deadline.

Your answers may be typed or handwritten. Please use the template provided with the assignment to answer your questions. Make sure that it is legible (for example, don't use very light pencil or ink if it is handwritten, or font size smaller than 11 if it is typed) or marks may be deducted.

You are to submit a SINGLE pdf file, where each page is A4 size. Do not submit files in other formats. If you submit multiple files, only the last submitted file will be graded.

Late submission will NOT be accepted. We have set the closing time of submission to slightly after 1pm to give you a few minutes of grace, but in your mind, you should treat **1pm** as the deadline. If you think you might be too busy on the day of the deadline, please submit earlier. Also, avoid submitting in the last few minutes, as the system may get sluggish due to overload and you will miss the deadline, and we will not extend the deadline for you.

Note the following:

- Name your pdf file with your Student Number. Your student number begins with 'A' (eg: A0234567X). (Do not mix up your student number with your NUSNET-id which begins with 'e'.)
- At the top of the first page of your submission, write your Name and Tutorial Group.
- To keep the submitted file short, please submit your answers without including the questions.
- As this is an assignment given well ahead of time, we expect you to work on it early. You should submit polished work, not answers that are untidy or appear to have been done in a hurry, for example, with scribbling and cancellation all over the places. Marks may be deducted for untidy work.
- It is always good to be clear and leave no gap so that the marker does not need to make guesswork on your answer. When in doubt, the marker would usually not award the mark.
- Do not use any methods that have not been covered in class. When using a theorem or result that has appeared in class (lectures or tutorials), please quote the theorem number/name, the lecture and slide number, or the tutorial number and question number in that tutorial, failing which marks may be deducted. Remember to use numbering and give justification for important steps in your proof, or marks may be deducted.

To combine all pages into a single pdf document for submission, you may find the following scanning apps helpful if you intend to scan your handwritten answers:

for Android: https://fossbytes.com/best-android-scanner-apps/

for iphone: https://www.switchingtomac.com/tutorials/ios-tutorials/the-best-ios-scanner-apps-to-scan-documents-images/

If you need any clarification about this assignment, please do NOT email us or post on telegram, but post on the Canvas > Discussions> Assignments forum or on QnA "Assignment" topic so that everybody can read the answers to the queries.

Question 0. (Total: 2 marks)
Check that …

- you have submitted a pdf file with your Student Number as the filename. [1 mark]
- you have written both your name and tutorial group number (e.g.: T02) at the top of the first page of your file. (If you miss either one you will not be given any mark.) [1 mark]

Question 1. Propositional logic (Total: 8 marks)
(a) Draw a truth table to prove each of the following statements. For truth tables, you may use **T** for true and **F** for false.

(i) $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$ [2 marks]

(ii) $(q \vee p) \wedge (r \vee p) \wedge (r \vee \sim q) \equiv (q \vee p) \wedge (r \vee \sim q)$ [2 marks]

(b) Simplify the statement form below in no more than 10 steps. (This question checks that you apply the laws rigorously and cite them correctly, so we will be strict in our grading.)

- Make sure you do not skip any step, and every step must be justified by a law.
- You are allowed to use any result that you have proved in Q1(a) as a law for the derivation.
- You may use a single law multiple times in a single statement and cite it once. For instance, if you use commutative law two times in a single step you may cite it as: (commutative law * 2).
- Use **true** and **false** for tautology and contradiction respectively.

$$\sim((p \rightarrow q) \wedge (q \rightarrow r))$$

The simplified answer must be in the following format.

$$(\cdot \vee \cdot) \wedge (\cdot \vee \cdot)$$

[4 marks]

Question 2. Argument (Total: 5 marks)
Determine whether the following argument is valid or invalid. Let $P1, P2$ and $P3$ denote the premises with $p, q,$ and r as the statement variables:

$$P1 \equiv p \rightarrow \sim q$$
$$P2 \equiv \sim r \rightarrow p$$
$$P3 \equiv q$$
$$\therefore r \vee p$$

You should begin your answer by stating "**Valid**" or "**Invalid**", followed by your proof. You may refer to premises as P1, P2, P3 instead of their statement form. You may any laws of equivalence and rules of inference covered in the class in your solution. You are not allowed to show your truth table (partial or full) in your answer, though you may use truth table in your own rough.

Question 3. Sets (Total: 5 marks)

No working/justification is required for this question.

For $k \in \mathbb{N}$ the set A_k is defined as $A_k = \{n \in \mathbb{N} \mid 0 \leq n \leq 5k\}$ and the set B_k is defined as $B_k = \{n + k \mid n \in \mathbb{N}\}$. Write down answers for the following questions.

a. $A_2 \cap B_2 = X \cup Y$ where $X\ and\ Y$ are any mutually disjoint sets. Write down sets $X\ and\ Y$ in the set-roster notation. [1 mark]

b. What is the natural number s that makes the following statement true: $\forall k \in \mathbb{N}\ A_k \cup B_k = B_s$? [1 mark]

c. What is the cardinality of $A_k \cap A_{k+1}$? [1 mark]

d. What is the cardinality of $B_k \setminus B_m$ for any two natural numbers k and m? [2 marks]

Question 4. Predicate Logic (Total: 8 marks)

Let S denote the set of all students and T denote the set of all tutorial classes. Define the predicate $R(x, y)$ to mean "student $x \in S$ is registered for tutorial class $y \in T$". Express each of the following statements in the formal language using predicates and quantifiers. Usage of the uniqueness quantifier ($\exists!$) is not allowed.

(a) Every student is registered in at most one tutorial. [2 marks]

(b) Negation of the statement (a). You are not allowed to simply state $\sim StatementA$. [2 marks]

(c) There is a tutorial class with exactly one student registered in it. [2 marks]

(d) Negation of the statement (c).). You are not allowed to simply state $\sim StatementC$. [2 marks]

Question 5. Proof (6 marks)

Complete the following proof that proves the following statement:

$$\forall a, b, c \in \mathbb{Z}\ \ (a^2 + b^2 = c^2) \rightarrow (Even(a) \vee Even(b))$$

Predicate $Even(a)$ evaluates to true if a is an even number, otherwise it evaluates to false.

Proof:

1. Let $a, b\ and\ c$ be any three integers.
2. Assume both $a\ and\ b$ are odd integers such that $a^2 + b^2 = c^2$
3. ....

Question 6. Proof on Sets (6 marks)

Prove for any sets A and B: $A \subseteq B \leftrightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B)$.

Marks will be deducted for long, unclear, or illogical arguments.

=== End of paper ===