**CS1231S Assignment #2**

AY2025/26 Semester 1

**Deadline: Monday, 3 November 2025, 1:00pm**

**IMPORTANT: Please read the instructions below carefully.**

This is a graded assignment worth 10% of your final grade. There are **six questions** (an admin Q0 and six task questions Q1–5) with a total score of 40 marks. Please work on it by yourself, not in a group or discussion/in collaboration with anybody. Anyone found committing plagiarism (submitting other's work as your own, using AI to help, etc.), or sending your answers to others, or other forms of academic dishonesty, will be penalised with a straight zero for the assignment, and reported to the school. Please see SoC website "Preventing Plagiarism" https://www.comp.nus.edu.sg/cug/plagiarism/.

You must submit your assignment to your respective tutorial group folder in **Canvas > Assignments > Assignment 2 submissions** before the deadline.

Your answers may be typed or handwritten. Make sure that it is legible (for example, don't use very light pencil or ink if it is handwritten, or font size smaller than 11 if it is typed) or marks may be deducted.

You are to submit **a SINGLE pdf file**, where each page is A4 size. Do not submit files in other formats. If you submit multiple files, only the last submitted file will be graded.

**Late submission will NOT be accepted**. We have set the closing time of submission to slightly after 1pm to give you a few minutes of grace, but in your mind, you should treat **1pm** as the deadline. Please submit early and avoid last minute work, as the system may get sluggish due to overload and you may miss the deadline, and we will not extend the deadline for you.

Note the following:

- Please use the template file provided. Rename the file with your Student Number (Axxxxxxxx.pdf).
- Note that if you make multiple submissions, Canvas will append a version number to your filename.
- At the top of the first page of your submission, write your Name and Tutorial Group.
- As this is an assignment given well ahead of time, we expect you to work on it early. You should submit polished work, not answers that are untidy or appear to have been done in a hurry, for example, with scribbling and cancellation all over the places. Marks may be deducted for untidy work.
- It is always good to be clear and leave no gap so that the marker does not need to make guesswork on your answer. When in doubt, the marker would usually not award the mark.
- Do not use any methods that have not been covered in class. When using a theorem or result that has appeared in class (lectures, tutorials, assignment), please quote the theorem number/name, the lecture and slide number, or the tutorial number and question number in that tutorial, failing which marks may be deducted. Remember to use numbering and give justification for important steps in your proof, or marks may be deducted.
- Check that your submitted file is complete. Common mistakes: Submitting a blank file, a file with missing pages, etc.

To combine all pages into a single pdf document for submission, you may find the following scanning apps helpful if you intend to scan your handwritten answers:

for Android: https://fossbytes.com/best-android-scanner-apps/

for iphone: https://www.switchingtomac.com/tutorials/ios-tutorials/the-best-ios-scanner-apps-to-scan-documents-images/

If you need any clarification about this assignment, please do NOT email us or post on telegram, but post on QnA "Assignment #2" topic so that everybody can read the answers to the queries.

**Question 0.** (Total: 2 marks)

Check that …

- you have submitted a pdf file with your Student Number as the filename. [1 mark]
- you have written both your name and tutorial group number (eg: T02A) at the top of the first page of your file. (If you miss either one you will not be given any mark.) [1 mark]

For questions on mathematical induction, follow the format we demonstrated in class, that is, define the predicate, state the basis step, state the inductive hypothesis explicitly, and work out the inductive step, with justification for important steps.

**Question 1.** (Total: 8 marks)

There is an alternative definition of inverse function for functions on a single set that could be useful:

| |
|---|
| Let $f : X \to X$ and $g : X \to X$ be two functions. Then $g$ is the inverse function of $f$ if and only if $g\big(f(x)\big) = x = f\big(g(x)\big)$ for all $x \in X$. |

You may use the above definition (optionally) wherever appropriate here.

(a) Let $A$ be a non-empty set and $f : A \to A$ a function.
Suppose there exists some function $g : A \to A$ such that $g \circ f = id_A$, then $f$ must be surjective. Is this statement true? Answer "true" or "false, then prove or disprove it. [2 marks]

(b) Let $A = \mathbb{R} \setminus \{0\}$ and define

$$f : A \to \mathbb{R}, \qquad f(x) = \frac{|x|-1}{x}$$

$|x|$ is the absolute value of $x$, defined as:

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Is $f$ a bijection? Answer "yes" or "no", then prove or disprove it. [3 marks]

(c) Let $D = \{(x, y) \in \mathbb{R}^2 : x \neq 1\}$ and define

$$g : D \to D, \qquad g\big((x, y)\big) = \left(x, y + \frac{x}{1-x}\right)$$

Is $g$ a bijection? Answer "yes" or "no", then prove or disprove it. [3 marks]

**Question 2.** (6 marks)

Using mathematical induction, prove that $\mathbf{15 \mid 2^{4n} + 31^n - 2}$ for all positive integers $n$.

**Question 3.** (6 marks)
Define a set $S$ recursively as follows:

(1) $\varepsilon \in S$. (base clause)

(2) If $s \in S$, then $xs \in S$, $sy \in S$, and $s^{-1} \in S$. (recursion clause)

(3) Membership for $S$ can always be demonstrated by (finitely many) successive applications of clauses above. (minimality clause)

Note that $s^{-1}$ denotes the reverse of $s$. That is, given any string $s = a_0a_1a_2 \cdots a_{m-2}a_{m-1} \in S$ of length $m$, $s^{-1} = a_{m-1}a_{m-2} \cdots a_2a_1a_0$.

Let $A = \{x, y\}$. Using mathematical induction, prove that $A^* = S$.

(Note that $A^*$ is the set of strings over $A$. Hint: use induction on the length of string.)

**Question 4.** (Total: 8 marks)
Let $R$ be a countably infinite subset of $\mathbb{R}$. A set $T$ is recursively defined as follows:

(1) For all $r \in R, r \in T$. (base clause)

(2) If $s \in T$ and $r \in R$, then $s + r \in T$. (recursion clause)

(3) Membership for $T$ can always be demonstrated by (finitely many) successive applications of clauses above. (minimality clause)

For each of the following statements, state whether it is "true" or "false", followed by a proof or counterexample. Remember to quote an appropriate theorem/result to justify an important step.

(a) $T$ is countably infinite for all choices of $R$. [5 marks]

(b) $T \setminus R$ is countably infinite for all choices of $R$. [3 marks]

**Question 5.** (Total: 10 marks)

(a) Find the number of integers between 3000 and 6000 inclusive in which no digits are repeated in each of the following independent cases. You do not need to show your working. [2 marks]

(i) There are no restrictions.

(ii) The integers are divisible by 10.

(b) 20 women and 9 men are to be seated in a straight row of 29 seats. One particular woman is Pauline, and two particular men are Pablo and Pallon. Find the number of arrangements in each of the following independent cases. You do not need to show your working. [4 marks]

(i) There are no restrictions.

(ii) Pablo and Pallon are seated next to each other.

(iii) Pauline is at the centre and both Pablo and Pallon are next to her.

(iv) Any two of Pauline, Pablo and Pallon are next to each other.

As the actual answers are very large numbers, you may write your answers in terms of factorial and $P(n, k)$, for example, $12! \times 20!$

(c) Find the number ways to seat 6 people around 2 identical circular tables such that no table is empty. You are to show your working in full. For your working, you may call the tables $A$ and $B$. [4 marks]