

CS1231S Assignment #1

AY2026/27 Semester 1

Deadline: Monday, 14 September 2026, 1:00pm

IMPORTANT: Please read the instructions below carefully.

This is a graded assignment worth 7% of your final grade. There are **four questions** with a total score of 21 marks. Please work on it by yourself, not in a group or discussion/in collaboration with anybody. Anyone found committing plagiarism (submitting other's work as your own, submitting an AI answer, or sending your answers to others, or other forms of academic dishonesty, will be penalised with a straight zero for the assignment, and reported to the school. Please see SoC website "Preventing Plagiarism" <https://www.comp.nus.edu.sg/cug/plagiarism/>.

You must submit your assignment to **Canvas > Assignments > Assignment 1 submission** before the deadline.

Your answers may be typed or handwritten. Make sure that it is legible (for example, don't use very light pencil or ink if it is handwritten, or font size smaller than 11 if it is typed) or marks may be deducted.

You are to submit a SINGLE pdf file, where each page is A4 size. Do not submit files in other formats. If you submit multiple times, only the last submitted file will be graded.

Late submission will NOT be accepted. We have set the closing time of submission to slightly after 1pm to give you a few minutes of grace, but in your mind, you should treat **1pm** as the deadline. If you think you might forget, please submit earlier. Also, avoid submitting in the last few minutes, as the system may get sluggish due to overload and you will miss the deadline, and we will not extend the deadline for you.

Note the following:

- An Answer Sheet is provided on Canvas. Download it and write/type your answers into it and submit it as a pdf file. Name your pdf file with your Student Number (which begins with 'A' and not with 'e' which is your NUSNET-id).
- Note that if you submit multiple times, Canvas will append some number to your filename; that's fine.
- You are not to write beyond the boxes provided in the Answer Sheet or add additional pages.
- As this is an assignment given well ahead of time, we expect you to work on it early. You should submit polished work, not answers that are untidy or appear to have been done in a hurry, for example, with scribbling and cancellation all over the places. Marks may be deducted for untidy work.
- It is always good to be clear and leave no gap so that the marker does not need to make guesswork on your answer. When in doubt, the marker would usually not award the mark.
- Do not use any methods that have not been covered in class. When using a theorem or result that has appeared in class (lectures or tutorials), please quote the theorem number/name, the lecture and slide number, or the tutorial number and question number in that tutorial, failing which marks may be deducted. Remember to use numbering and give justification for important steps in your proof, or marks may be deducted.

To combine all pages into a single pdf document for submission, you may find the following scanning apps helpful if you intend to scan your handwritten answers:

for Android: <https://fossbytes.com/best-android-scanner-apps/>

for iphone: <https://www.switchingtomac.com/tutorials/ios-tutorials/the-best-ios-scanner-apps-to-scan-documents-images/>

If you need any clarification about this assignment, please do NOT email us or post on telegram, but post on QnA "Assignments" topic so that everybody can read the answers to the queries.

Question 1. Argument (5 marks)

Given the following argument on statement variables p, q , and r , determine whether the argument is valid or invalid.

$p \rightarrow q$	(Premise P1)
$\sim r \rightarrow \sim q$	(Premise P2)
p	(Premise P3)
$\therefore \sim q \vee r$	(Conclusion)

You should begin your answer by stating “**Valid**” or “**Invalid**”, followed by your proof, showing step by step. You may refer to the premises as P1, P2 and P3. Do not show your truth table (partial or full) in your answer, though you may use truth table in your own rough work to help you derive your proof.

Remember to justify each step with an equivalence law or a rule of inference (yes, we want you to explicitly state them for now, for example, “contrapositive of P1”, “transitivity on lines 3 and 4”, etc.).

Answer. The argument is valid.

- | | | |
|----------------------|-----------------------------|---|
| 1. $q \rightarrow r$ | (contrapositive of P2) | Explanation: $(\sim r \rightarrow \sim q) \equiv (q \rightarrow r)$ |
| 2. $\sim q \vee r$ | (implication law on line 1) | Explanation: $q \rightarrow r \equiv \sim q \vee r$ |

Note that **explanations** are not required by students.

Other (longer) answers:

- | | | |
|----------------------|---------------------------------|---|
| 1. q | (modus ponens on P1 and P3) | Explanation: $(p \rightarrow q) \wedge p \rightarrow q$ |
| 2. $q \rightarrow r$ | (contrapositive of P2) | Explanation: $(\sim r \rightarrow \sim q) \equiv (q \rightarrow r)$ |
| 3. r | (modus ponens on lines 2 and 1) | Explanation: $(q \rightarrow r) \wedge q \rightarrow r$ |
| 4. $\sim q \vee r$ | (generalization on line 3) | Explanation: $r \rightarrow \sim q \vee r$ |

Alternative proof:

- | | |
|----------------------|---------------------------------|
| 1. $q \rightarrow r$ | (contrapositive of P2) |
| 2. $p \rightarrow r$ | (transitivity on P1 and line 1) |
| 3. r | (modus ponens on line 2 and P3) |
| 4. $\sim q \vee r$ | (generalization on line 3) |

Question 2. Sets (Total: 5 marks)

No working/justification is required for this question.

- (a) For each $i \in \{1,2,3,4\}$, define $A_i = \{ik + 2 : k \in \{0,1,2,3,4\}\}$. Write down B_2, B_3, B_4 in **roster notation** with the following properties, where B_1, B_2, B_3, B_4 are mutually disjoint sets:

$$B_1 = A_1,$$

$$B_1 \cup B_2 = A_1 \cup A_2,$$

$$B_1 \cup B_2 \cup B_3 = A_1 \cup A_2 \cup A_3,$$

$$B_1 \cup B_2 \cup B_3 \cup B_4 = A_1 \cup A_2 \cup A_3 \cup A_4.$$

Do not use ellipses ("...") in your answer.

[3 marks]

- (b) Let $A = \{-4, -2, -1, 1, 2, 4\}$, $B = \{0, 3, 15\}$ and $C = \{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$.

Determine whether the statement below is true or false. Start your answer by writing "True" or "False", followed by your proof or counterexample.

[2 marks]

$$\forall x \in C ((x \in A) \leftrightarrow (x^2 - 1 \in B))$$

You may have observed that the space given in the Answer Sheet for this part is small, a hint that you are to provide a succinct and concise answer.

Answers

- (a) $B_2 = \{8, 10\}$, $B_3 = \{11, 14\}$, $B_4 = \{18\}$.

(For reference: $A_1 = \{2, 3, 4, 5, 6\}$, $A_2 = \{2, 4, 6, 8, 10\}$, $A_3 = \{2, 5, 8, 11, 14\}$, $A_4 = \{2, 6, 10, 14, 18\}$.)

- (b) True.

$$(x^2 - 1 \in B) \equiv (x^2 - 1 \in \{0, 3, 15\}) \equiv (x^2 \in \{1, 4, 16\}) \equiv (x \in \{-1, 1, -2, 2, -4, 4\}) \equiv (x \in A).$$

Question 3. Proof (5 marks)

Prove the following statement:

$$\forall a, b \in \mathbb{Z} (a \geq 2 \rightarrow a \nmid b \vee a \nmid (b + 1))$$

(You may cite closure of integers under subtraction if you find this useful.)

Proof (by contraposition):

1. Let a and b be arbitrary integers.
2. Suppose $\sim(a \nmid b \vee a \nmid (b + 1))$,
 - 2.1. Then we have $\sim(a \nmid b) \wedge \sim(a \nmid (b + 1))$ (by De Morgan's law)
 - 2.2. Simplifying, we have $(a \mid b) \wedge (a \mid (b + 1))$ (by double negative law)
 - 2.3. Since $a \mid b$, there exists an integer k such that $b = ka$ (by definition of divisibility)
 - 2.4. Similarly, since $a \mid (b + 1)$, there exists an integer k' such that $b + 1 = k'a$ (by definition of divisibility)
 - 2.5. Therefore, $k'a - ka = (b + 1) - b = 1$ (basic algebra)
 - 2.6. Thus $(k' - k)a = 1$ (basic algebra)
 - 2.7. Since $k' - k \in \mathbb{Z}$ (by closure of integers under subtraction), we have $a \mid 1$ (by definition of divisibility)
 - 2.8. Since the only divisors of 1 are 1 and -1, it is true that $a < 2$.
3. Since the contrapositive holds, the statement is true.

Question 4. Proof on sets (Total: 6 marks)

In tutorial 3, defined the set operator $\oplus$ as follows:

$$A \oplus B = (A \setminus B) \cup (B \setminus A).$$

Prove the following:

- (a) $\oplus$ is commutative. [1 mark]
- (b) $\oplus$ has an identity element, i.e., there exists a set I such that for all sets A , $I \oplus A = A$. [3 marks]
- (b) Every set A has an inverse A^{-1} under $\oplus$ such that $A \oplus A^{-1}$ equals the identity element. [2 marks]

Answers

(a)

- 1. $A \oplus B = (A \setminus B) \cup (B \setminus A)$ (by the definition of $\oplus$)
- 2. $= (B \setminus A) \cup (A \setminus B)$ (by the commutativity of union)
- 3. $= B \oplus A$ (by the definition of $\oplus$)
- 4. Therefore, $\oplus$ is commutative.

(b)

- 1. We show the identity element is $\emptyset$. Let A be any set.
- 2. $A \oplus \emptyset = (A \setminus \emptyset) \cup (\emptyset \setminus A)$ (by the definition of $\oplus$)
- 3. $= (A \cap \overline{\emptyset}) \cup (\emptyset \cap \bar{A})$ (by the definition of $\setminus$)
- 4. $= (A \cap U) \cup (\emptyset \cap \bar{A})$ (complement of $\emptyset$)
- 5. $= A \cup (\emptyset \cap \bar{A})$ (by the identity law)
- 6. $= A \cup (\bar{A} \cap \emptyset)$ (by the commutative law)
- 7. $= A \cup \emptyset$ (by the universal bound law)
- 8. $= A$ (by the identity law)
- 9. Therefore, $\emptyset$ is the identity element for $\oplus$.

(c)

- 1. Let A be any set.
- 2. $A \oplus A = (A \setminus A) \cup (A \setminus A)$ (by the definition of $\oplus$)
- 3. $= (A \cap \bar{A}) \cup (A \cap \bar{A})$ (by the definition of $\setminus$)
- 4. $= \emptyset \cup \emptyset$ (by the complement law)
- 5. $= \emptyset$ (by the idempotent law)
- 6. Therefore, every set has an inverse that is the set itself.

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