CS1231S Discrete Structures

Assignment #2 Answer Sheets

AY2026/27 Semester 1

Student number: A Tutorial group:

INSTRUCTIONS

1. Write your **Student Number** and **tutorial group number** (eg: T03B) above.
2. Submit only this document as a pdf file and write your answers inside the boxes provided.

Q1. (Total: 4 marks)

(a) [1] Is f injective?

(b) [1] Is f surjective?

(c) [1] Fixed points:

(d) [1] Periodic points of period 2:

Q2. (Total: 5 marks)

(a) [1] Is f injective?

(b) [1] $(f \circ f)(abaa)$

(c) [1] Is $f \circ f$ injective?

(d) [2] Is g surjective?

Q3. (Total: 6 marks) Write your answer for Q3 on only this page.

(a) Proof of existence for positive integers n:

1. Let $P(n) \equiv (\forall d \in \mathbb{Z}^+ \exists q, r \in \mathbb{Z}$ such that $n = dq + r$, where $0 \leq r < d)$, for any $n \in \mathbb{N}$.
2. Basis step:

3. Indiuctive step: Assume $P(k)$ is true for any $k \in \mathbb{N}$, i.e. $\forall d \in \mathbb{Z}^+ \exists q, r \in \mathbb{Z}$ such that $k = dq + r$, where $0 \leq r < d$.

4. Therefore, $\forall n \in \mathbb{N}\ P(n)$ is true by mathematical induction.

(b) Proof of existence for integers n:

1. $0 = 0q + 0$. So $P(0)$ is true. It suffices to show that $P(n)$ is true for all negative integers n.
2.

3. Therefore, in all cases $P(n)$ is true for $n \in \mathbb{Z}^-$.
4. With part (a), $P(n)$ is true for $n \in \mathbb{Z}$.

Q4. (Total: 6 marks)

(a)
[1]

(b)
[2]

(c)
[3]

=== END OF PAPER ===