NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

MID-TERM TEST
AY2025/26 Semester 1

CS1231S — DISCRETE STRUCTURES

8 October 2025 Time Allowed: 1 hour 30 minutes

INSTRUCTIONS

1. This assessment paper contains **NINETEEN (19)** questions in **TWO (2)** parts and comprises **EIGHT (8)** printed pages.
2. This is an **OPEN BOOK** assessment.
3. Printed/written materials are allowed. Apart from calculators, electronic devices are not allowed.
4. Answer **ALL** questions on the **ANSWER SHEETS** provided.
5. Shade your Student Number and MCQ answers **ONLY with a pencil** (2B or above).
6. You may write your answers in pen or pencil, though we advise you to use pencil in case you need to erase and rewrite your answers. Write legibly, or marks may be deducted.
7. You are to write within the space provided. No extra pages should be submitted as it will jam the scanning machine.
8. The maximum mark of this assessment is 50.
9. Do not start writing or flip over this page until you are told to do so.

——— END OF INSTRUCTIONS ———

Part A: Multiple Choice Questions (Total: 30 marks)

Each multiple-choice question (MCQ) is worth two marks and has exactly **one** correct answer.

1. Which of the following celebrated name is not involved with the modern set theory?
   A. Georg Cantor
   B. Bertrand Russell
   C. Ernst Zermelo
   D. Carl Gauss
   E. Abraham Fraenkel

2. Let D be the set of all objects in LT11. We define the following predicates on objects in D: $Red(x)$ evaluates to true if x is red and $Flying(x)$ evaluates to true if x is flying. Consider the statement:

$$\forall x \in D \left(Red(x) \wedge Flying(x) \right)$$

   Which of the following option is false?
   A. If the statement is true, then LT11 only contains red objects.
   B. If the statement is true, then LT11 only contains flying objects
   C. The statement is true if LT11 does not contain any objects.
   D. If the statement is false, LT11 contains at least one object.
   E. At least one of the options (A), (B), (C), (D) is incorrect.

3. Given the statement form below where a, b, c are statement variables,

$$\big(a \wedge \sim(b \wedge c)\big) \vee \Big(\sim a \wedge \big(b \vee (\sim b \wedge c)\big)\Big)$$

   Which of the following is logically equivalent to the above?
   A. $(\sim a \wedge b) \vee (a \wedge \sim c) \vee (\sim b \wedge c)$
   B. $(\sim a \wedge c) \vee (b \wedge \sim c) \vee (\sim b \wedge a)$
   C. $\sim\big((a \leftrightarrow b) \wedge (a \leftrightarrow c)\big)$
   D. $(\sim a \vee \sim b \vee \sim c) \vee (a \vee b \vee c)$
   E. All options are equivalent to each other.

4. Consider a set $A = \{1, 2, 3\}$. How many relations defined on set A are equivalence relations?
   A. 3
   B. 4
   C. 5
   D. 7
   E. None of options (A), (B), (C), (D) are correct.

5. Please refer to the diagram in the appendix. Consider the following statements.

   (i) There is a genre such that every female reader reads at least one title in it.

   (ii) There is a genre such that some male reader reads all titles in it.

   (iii) There is a female reader x such that for all male readers y there is a title read by both reader x and reader y.

   Choose one the right options below.

   A. Exactly one of (i), (ii) and (iii) is true.
   B. (i) and (ii) are true but (iii) is not true.
   C. (i) and (iii) are true but (ii) is not true.
   D. (ii) and (iii) are true but (i) is not true.
   E. None of options (A), (B), (C), (D) are correct.

6. Let n be any natural number. Which of the following statements are true when n is a perfect square?

   (i) n is a prime number.

   (ii) Number of distinct factors of n is odd.

   (iii) For any prime number p, if $p \mid n$ then $p^2 \mid n$.

   A. Only (i).
   B. Only (ii).
   C. Only (iii)
   D. Both (ii) and (iii)
   E. Both (i) and (iii)

7. Let A, B, C be three sets such that any two of them have exactly one element in common. Let $D = \mathcal{P}(A) \cap \mathcal{P}(B) \cap \mathcal{P}(C)$. What is the set of possible cardinalities of D?

   A. $\{1\}$
   B. $\{1, 2\}$
   C. $\{0, 1\}$
   D. $\{0, 1, 2\}$
   E. None of options (A), (B), (C), (D) are correct.

8. Assume that A has twice as many elements as its subset B, that B has two elements more than C. If B, C and D forms a partition over A, how many elements are there in D?

   A. 0
   B. 1
   C. 2
   D. 3
   E. Strictly more than 3

9. Consider the sets $A = \{1, 2, 3\}$ and $B = \{2, 5\}$. Consider the following sets defined using these:

   (i) $S_1 = \mathcal{P}(A) \times \mathcal{P}(B)$

   (ii) $S_2 = \mathcal{P}(A \times B)$

   (iii) $S_3 = \{R: A \to B \mid R \; is \; a \; relation\}$.

   (iv) $S_4 = \{R: A \to B \mid R \; is \; a \; symmetric \; relation\}$.

   Which of the following options is correct regarding the following sets?

   A. $S_4 \subseteq S_3 \subseteq S_2$

   B. $(S_2 \subseteq S_1) \wedge (S_3 \subseteq S_4) \wedge (S_2 \subseteq S_3)$

   C. $(S_4 \subseteq S_3) \wedge (S_1 = S_2)$

   D. $(S_2 \subseteq S_1) \wedge (S_3 \subseteq S_4)$

   E. $S_1 \subseteq S_2 \subseteq S_3 \subseteq S_4$

10. Logical connectives can be viewed as a relation on the set $\mathcal{B} = \{true, false\}$. Consider relation R that embodies the logical implication $\to$, defined as: $\forall p, q \in \mathcal{B} \; \big(pRq \leftrightarrow (p \to q)\big)$. What is the inverse of the relation R?

    A. R.

    B. The inverse of the implication.

    C. The contrapositive of the implication.

    D. The converse of the implication.

    E. None of options (A), (B), (C), (D) are correct.

11. Consider a relation R defined on $\mathcal{P}(\mathbb{Z})$. It is defined as follows:

$$\forall A, B \in \mathcal{P}(\mathbb{Z}) \; \big((A, B) \in R \; \leftrightarrow |A \cap B| = 1\big).$$

    Which of the following statements is true regarding the following sets.

    A. R is not a symmetric relation.

    B. R is a symmetric relation.

    C. R is an equivalence relation.

    D. R is a partial-order relation.

    E. R is an anti-symmetric relation.

12. Which of the following statements is true regarding the following sets?

$$F = \{f : \{0,1\} \to \{0,1\} \mid f \text{ is an equivalence relation.}\}$$
$$G = \{g : \{0,1\} \to \{0,1\} \mid g \text{ is a partial order relation.}\}$$

A. $(|F| + 1) < |G|$
B. $(|F| + 1) = |G|$
C. $|F| = |G|$
D. $|F| = (|G| + 1)$
E. $|F| < (|G + 1|)$

13. Let X denote the domain of arguments. $Sound(x)$ evaluates to true if x is a sound argument, otherwise it is false. $Valid(x)$ evaluates to true if x is a valid argument, otherwise it is false. Which of the following statements are true?

I. $\forall x \in X \big(Sound(x) \to Valid(x)\big)$.
II. $\forall x \in X \big(Valid(x) \to Sound(x)\big)$.
III. $\exists x \in X \big(Sound(x) \wedge \sim Valid(x)\big)$.
IV. $\exists x \in X \big(\sim Sound(x) \wedge Valid(x)\big)$.

A. Only I
B. Only II
C. Only II and III
D. Only I and IV
E. All statements are true.

14. Let $\mathcal{R}$ denote the set of all reflexive relations on $A = \{1, 2, 3, 4\}$. Consider the following set:

$$\mathcal{F} = \{R \in \mathcal{R} : \forall x, y, z \in A \big((xRy \wedge xRz) \to (y = z)\big)\}$$

What is the cardinality of $\mathcal{F}$?

A. 0
B. 1
C. 2
D. 4
E. 16

15. Let $\sim$ be an equivalence relation defined on a non-empty set A. Which of the following statements are true?

    I. $\exists\, x \in (A/\sim)\ \forall\, y \in x\ \ (y \in A)$

    II. $\forall x \in A\ \ ([x]_{\sim} \subseteq A/\sim)$

    III. $|A| = \sum_{x \in A/\sim} |x|$

    A. Only I

    B. Only II

    C. Only II, and III

    D. Only I, and II

    E. Only I, and III

Part B (Total: 20 marks)

16. **Proof [Total: 6 marks]**

Let $Animal = \{cat, sheep, rabbit, penguin\}$ and $Colours = \{white, brown, grey, black\}$ be two sets. Consider the following statements:

(1) Every animal has at least one colour.
(2) An animal has a six-lettered name if and only if it has black colour.
(3) If an animal has the letter ``e'' then it has the colour white.
(4) If an animal contains a repeated letter then it does not have the colour grey.
(5) If an animal has the colour brown then it has the letter ``i''.
(6) The penguin has exactly two colours.
(7) The cat and the sheep have exactly one colour.
(8) There is an animal which has exactly two colours: brown and black.
(9) For every colour there is an animal which has this colour.

Assuming that the statements above are true, list out for each animal the set of colours it has.

17. **Relations [Total: 14 marks]**

(a) Given $S = \{(1,3),(3,6),(6,9),(9,12)\}$ and $R = \{(6,6),(9,9)\}$ be two relations. Write down the following in the set-roster notation:

(i) $(R \circ S) \setminus (S \circ R)$ (ii) $(R^{-1} \circ S^{-1}) \circ (S \circ R)$ [4 marks]

(b) Write down the following sets (that are subsets of $\mathbb{N}$) in the replacement notation (for part(i)) and in the set-builder notation (for part (ii)).

(i) $A = \{0,\ 1,\ 3,\ 7,\ 15,\ 31, \dots\}$ (ii) $B = \{3, 9, 15, 21, 27, \dots\}$ [2 marks]

(c) Consider $\mathcal{P}$ be the entire population on earth. We define two relations on this set.

$\forall\, x, y \in \mathcal{P}\ (xR_1y \ \leftrightarrow\ x \text{ and } y \text{ share biological father and biological mother})$
$\forall\, x, y \in \mathcal{P}\ (xR_2y \ \leftrightarrow\ x \text{ and } y \text{ share a biological parent})$

Answer following questions as **Yes/No** based on these relations. If the answer is **No**, please mention the property the relation violates.

(i) Is R_1 an equivalence relation?

(ii) Is R_2 an equivalence relation? [2 marks]

(d) Write down $\mathbb{Z}^+/\sim$ for the equivalence relation defined as follows: [2 marks]

$$\forall x, y \in \mathbb{Z}^+ \left(x \sim y \leftrightarrow\ \exists d \in \mathbb{Z}^+\ (d \mid x) \wedge (d \mid y)\right)$$

(e) Consider the poset $(\{\{a\},\{a,b\},\{a,c\},\{a,b,c\},\{a,b,c,d\},\{c,d\}\},\subseteq)$. [4 marks]

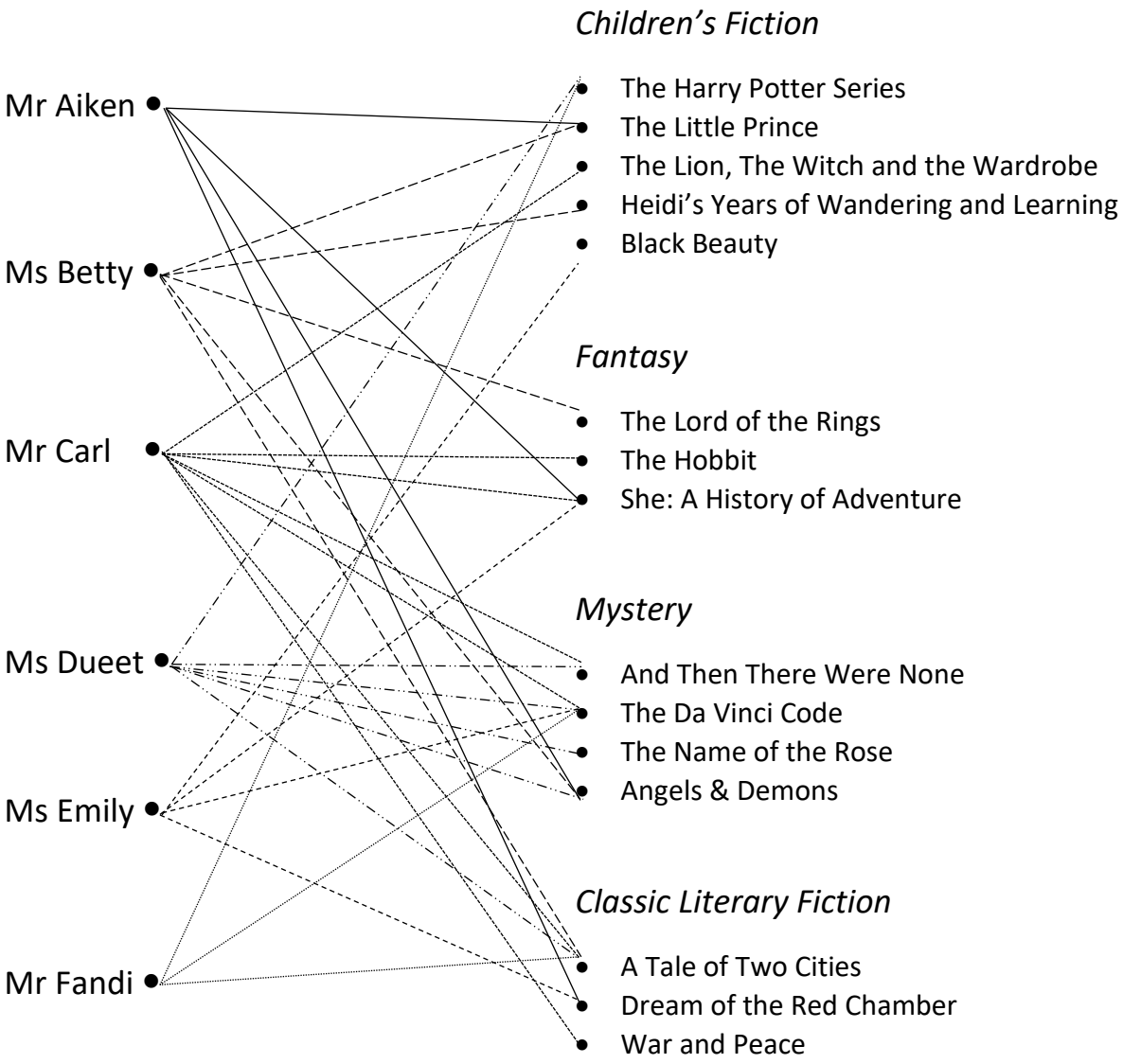
(i) Draw Hasse Diagram.

(ii) Is there a smallest element in the poset? Write 'yes' or 'no'. If yes, state the element.

(iii) Are there any maximal elements in the poset? Write 'yes' or 'no'. If yes, state all the maximal elements.

=== END OF PAPER ==

Appendix


Mr Aiken
Ms Betty
Mr Carl
Ms Dueet
Ms Emily
Mr Fandi
Children's Fiction
The Harry Potter Series
The Little Prince
The Lion, The Witch and the Wardrobe
Heidi's Years of Wandering and Learning
Black Beauty
Fantasy
The Lord of the Rings
The Hobbit
She: A History of Adventure
Mystery
And Then There Were None
The Da Vinci Code
The Name of the Rose
Angels & Demons
Classic Literary Fiction
A Tale of Two Cities
Dream of the Red Chamber
War and Peace