NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

MID-TERM TEST
AY2026/27 Semester 1

ANSWERS

CS1231S — DISCRETE STRUCTURES

7 October 2026 Time Allowed: 1 hour 30 minutes

INSTRUCTIONS

1. This assessment paper contains **EIGHTEEN (18)** questions in **TWO (2)** parts and comprises **SEVEN (7)** printed pages.
2. This is an **OPEN BOOK** assessment.
3. Printed/written materials are allowed. Apart from calculators, electronic devices are not allowed.
4. Answer **ALL** questions on the **ANSWER SHEET** provided.
5. Shade your Student Number and MCQ answers **ONLY with a pencil** (2B or above).
6. You may write your answers for Part B in pen or pencil, though we advise you to use pencil in case you need to erase and rewrite your answers. Write legibly, or marks may be deducted.
7. You are to write within the boxes provided in the Answer Sheet. No extra pages should be submitted as it will jam the scanning machine.
8. The maximum mark of this assessment is 50.
9. Do not start writing or flip over this page until you are told to do so.

——— END OF INSTRUCTIONS ———

1	C	2	D	3	A	4	A	5	B	6	B
7	B	8	D	9	C	10	B/E	11	E	12	C
13		14		15		16					

Part A: Multiple Choice Questions (Total: 24 marks)

Each multiple choice question (MCQ) is worth two marks and has exactly **one** correct answer.

1. Consider the following statements.

   (i) $\sim (p \leftrightarrow q) \wedge \sim (q \leftrightarrow r) \wedge \sim (r \leftrightarrow p)$
   (ii) $(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$
   (iii) $(p \wedge q) \vee (q \wedge r) \vee (r \wedge p)$
   (iv) $(p \vee q \vee r) \wedge (\sim p \wedge \sim q \wedge \sim r)$
   (v) $(p \wedge \sim p) \vee (q \wedge \sim q) \vee (r \wedge \sim r)$

   What is the largest number of statements from above that are equivalent to one another?

   A. 1.
   B. 2.
   C. 3.
   D. 4.
   E. 5.

   Answer: C (1 ,4 and 5 are equivalent to false)

2. Let $B = \{true, false\}$. Consider the following sets and their cardinalities.

   (i) $a = |(p, q) \in B \times B : p \wedge q \equiv true\}|$
   (ii) $b = |(p, q) \in B \times B : p \vee q \equiv true\}|$
   (iii) $c = |(p, q) \in B \times B : p \leftrightarrow q \equiv true\}|$

   Which of the following statement is correct for the set $N = \{a, b, c\}$?

   A. $\exists n \in N\ n = 0$.
   B. $\forall n \in N\ \exists m \in N\ m = n + 1$.
   C. $\exists m \in N\ \forall n \in N\ m = n + 1$.
   D. $\exists! n \in N\ Even(n)$.
   E. None of options (A), (B), (C), (D) are correct.

   Answer: D (N = {1, 3, 2})

3. Which of the following is/are always true for any finite sets A and B?
   (Note: $\mathcal{P}(X)$ is the power set of X.)

   (i) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$
   (ii) $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$
   (iii) $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$
   (iv) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

   A. Only (ii).
   B. Only (iv).
   C. Only (i) and (ii).
   D. Only (iii) and (iv).
   E. None of options (A), (B), (C), (D) are correct.

Answer: A
Counter-example for (i), (iii) and (iv). Let $A = \{1\}, B = \{2\}$.
Then $\mathcal{P}(A \cup B) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$ but $\mathcal{P}(A) \cup \mathcal{P}(B) = \{\emptyset, \{1\}, \{2\}\}$ and $\mathcal{P}(A) \cap \mathcal{P}(B) = \{\emptyset\}$.

4. Let $A = \{A_1, A_2, A_3, \cdots, A_m\}$ and $B = \{B_1, B_2, B_3, \cdots, B_n\}$ be partitions of a non-empty set S. Consider

$$C = \{A_i \cap B_j : (1 \leq i \leq m) \wedge (1 \leq j \leq n) \wedge (A_i \cap B_j \neq \emptyset)\}.$$

Which of the following statements is correct?

A. C is always a partition of S.
B. C is a partition of S only if $A = B$.
C. C is a partition of S only if $m = n$.
D. C needs not be a partition of S because two intersections may overlap.
E. None of options (A), (B), (C), (D) are correct.

Answer: A
C is a common refinement of A and B.
Suppose P and Q are both partitions of a set S. We say that P is a refinement of Q if every component of P is a subset of some component of Q.

5. Suppose R and S are equivalence relations on a nonempty set A. Which condition is sufficient to guarantee that $R \cup S$ is also an equivalence relation on A?

A. $R \cap S \neq \emptyset$
B. $R \subseteq S$ or $S \subseteq R$
C. $R \neq S$
D. There is no way for $R \cup S$ to be an equivalence relation on A.
E. None of options (A), (B), (C), (D) are correct.

Answer: B

6. Which of the following statements is/are always true for all finite sets A and B?

(i) $A \times B = B \times A$
(ii) $|A \times B| = |A| \times |B|$
(iii) If $A \neq B$, then $A \times B \neq B \times A$.
(iv) If $A \times B = \emptyset$, then $(A = \emptyset) \vee (B = \emptyset)$.

A. Only (iv).
B. Only (ii) and (iv).
C. Only (i), (ii) and (iv).
D. Only (ii), (iii) and (iv).
E. None of options (A), (B), (C), (D) are correct.

Answer: B
(iii) Let $A = \emptyset, B = \{1\}$. Then $A \times B = \emptyset = B \times A$.

7. Suppose R is an equivalence relation on a nonempty set A, and $a, b \in A$.
Which of the following statements is/are not necessarily true?

(i) $a \; R \; b \to [a] = [b]$
(ii) $[a] \cap [b] \neq \emptyset \to [a] = [b]$
(iii) $a \neq b \to [a] \neq [b]$

A. Only (i).
B. Only (iii).
C. Only (i) and (ii).
D. Only (ii) and (iii).
E. None of options (A), (B), (C), (D) are correct.

Answer: B

8. For each integer $n \geq 2$, define

$$A_n = \{k \in \mathbb{Z}^+ : n \mid k\} \setminus \{n\}.$$

Which of the following is correct?

(Note: Composite number is defined in Lecture 4:
An integer n is composite iff $n = rs$ for some integers r and s with $1 < r < n$ and $1 < s < n$.)

A. $\bigcup_{n=2}^{\infty} A_n = \mathbb{Z}^+$
B. $\bigcup_{n=2}^{\infty} A_n = \mathbb{Z}^+ \setminus \{1\}$
C. $\bigcup_{n=2}^{\infty} A_n = \mathbb{Z}^+ \setminus \{1,2\}$
D. $\bigcup_{n=2}^{\infty} A_n$ is the set of composite integers
E. None of options (A), (B), (C), (D) are correct.

Answer: D

9. Let R be a relation on a nonempty set A. Suppose R is both symmetric and antisymmetric. What can be concluded?

A. R must be empty.
B. R must be the identity relation $\{(a, a) : a \in A\}$.
C. $\forall x, y \in A \left((x, y) \in R \to x = y\right)$.
D. R must be reflexive.
E. None of options (A), (B), (C), (D) are correct.

Answer: C

10. Which of the following identities is/are NOT true for all sets A, B, C?

(i) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

(ii) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

(iii) $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$

(iv) $A \cup (B \setminus C) = (A \cup B) \setminus (A \cup C)$

A. Only (i).

B. Only (iv).

C. Only (i) and (ii).

D. Only (iii) and (iv).

E. None of options (A), (B), (C), (D) are correct.

Answer: B

(i) is true. $A \setminus (B \cup C) = A \cap \overline{(B \cup C)} = A \cap (\bar{B} \cap \bar{C}) = (A \cap \bar{B}) \cap (A \cap \bar{C}) = (A \setminus B) \cap (A \setminus C)$.

(ii) is true. $A \setminus (B \cap C) = A \cap \overline{(B \cap C)} = A \cap (\bar{B} \cup \bar{C}) = (A \cap \bar{B}) \cup (A \cap \bar{C}) = (A \setminus B) \cup (A \setminus C)$.

(iii) is true. $A \cap (B \setminus C) = A \cap (B \cap \overline{C}) = (A \cap B) \cap \bar{C}$;

$(A \cap B) \setminus (A \cap C) = (A \cap B) \cap (\bar{A} \cup \bar{C}) = \big((A \cap B) \cap \bar{A}\big) \cup \big((A \cap B) \cap \bar{C}\big) = (A \cap B) \cap \bar{C}$.

(iv) is not true. Let $A = \{1\}, B = \{2\}, C = \emptyset$. Then $A \cup (B \setminus C) = \{1,2\}$ but $(A \cup B) \setminus (A \cup C) = \{2\}$

11. Consider a relation $R = \{(1,1), (2,3)\}$ over the domain $\{1, 2, 3\}$. Which of the following options is correct?

A. R is an equivalence relation.

B. $R \circ R$ is an equivalence relation.

C. $R \circ R^{-1}$ is an equivalence relation.

D. $R^{-1} \circ R$ is an equivalence relation.

E. None of options (A), (B), (C), (D) are correct.

Answer: E

12. Consider the following relations defined on $\mathbb{N} \times \mathbb{N}$:

$$\forall (a,b),(c,d) \in \mathbb{N} \times \mathbb{N} \ \big((a,b) R_1 (c,d) \leftrightarrow a \leq c\big)$$

$$\forall (a,b),(c,d) \in \mathbb{N} \times \mathbb{N} \ \big((a,b) R_2 (c,d) \leftrightarrow (a \leq c) \wedge (b \geq d)\big)$$

Which of the following options is correct about these relations?

A. Both R_1 and R_2 are not partial order relations.

B. R_1 is a partial order relation but R_2 is not a partial order relation.

C. R_1 is not a partial order relation but R_2 is a partial order relation.

D. Both R_1 and R_2 are partial order relations.

E. Either R_1 or R_2 is a total order relation.

Answer: C (R_1 is not anti-symmetric. R_2 is a partial order relation but not a total order. (1, 2) and (2, 3) are not comparable under R_2)

Part B: Short Answer Questions (Total: 26 marks)

13. (a) [1 mark] Rewrite $\sim(\sim p \to q)$ using only negation and conjunction. Only write the final answer.

Answer: $\sim p \wedge \sim q$

(b) [4 marks] The following is a proof by contradiction for the following argument:

$$\begin{array}{ll} p \to q & (Premise\ P1) \\ \sim r \vee s & (Premise\ P2) \\ p \vee r & (Premise\ P3) \\ \hline \sim q \to s & (Conclusion) \end{array}$$

Fill in the missing blanks below in the Answer Sheet to complete the proof.

	Statement	Justification
1	$\sim(\sim q \to s)$	Assumption
2	$\sim q \wedge \sim s$	From Q13(a)
3	$\sim s$	Specialisation on 2
4	$\sim r \vee s$	Premise P2
5	$\sim r$	Elimination on 3, 4
6	$p \to q$	Premise P1
7	$\sim q$	Specialisation on 2
8	$\sim p$	Modus tollens on 6, 7
9	$p \vee r$	Premise P3
10	r	Elimination on 8, 9
11	$r \wedge \sim r$	Conjunction Introduction
12	$\boldsymbol{false}$	Negation Law on 11
13	$\therefore \sim q \to s$	Conclusion using proof by Contradiction

14. [3 marks] Write the smallest possible set A in the set-roster notation that satisfies the following conditions: $\{\{1\}, 2\} \subseteq A$ and $\{\{1\}, 2\} \in A$.

Answer: $A = \{\{1\}, 2, \{\{1\}, 2\}\}$. 1 mark for each correct element. -1 mark for each incorrect element

15. [6 marks] Let $A = \{2, 3, 6\}$. We define two relations R and S on $A \times A$ as follows:

$$(a, b)R(c, d) \leftrightarrow (a \leq b) \wedge (c \leq d) \wedge (ab = c + d)$$
$$(a, b)S(c, d) \leftrightarrow (a \leq b) \wedge (c \geq d) \wedge (a + b = cd)$$

(a) [2 marks] Write down R in set-roster notation.

(b) [4 marks] Write down $(S \circ R)^{-1}$ in set-roster notation.

Answer:

(a) $\boldsymbol{R = \{((2,2),(2,2)),((2,3),(3,3)),((2,6),(6,6)),((3,3),(3,6)),\}}$ **Marking scheme: 0.5 mark for each correct element, -0.5 mark for each incorrect element.**

(b) $\boldsymbol{S = \{((2,2),(2,2)),((3,3),(3,2)),((3,6),(3,3)),((6,6),(6,2))\}}$

$\boldsymbol{S \circ R = \{((2,2),(2,2)),((2,3),(3,2)),((2,6),(6,2)),((3,3),(3,3))\}}$

$\boldsymbol{(S \circ R)^{-1} = \{((2,2),(2,2)),((3,2),(2,3)),((6,2),(2,6)),((3,3),(3,3))\}}$

Marking scheme: 1 mark for each correct element, -1 mark for each incorrect element.

16. [6 marks] Define an equivalence relation $\sim$ on $\mathbb{Z}$ such that $x \sim y \leftrightarrow 4 \mid (7x - 3y)$.

(a) [2 marks] What is $[4]$? (Use replacement notation)

(b) [2 marks] What is $[-3]$? (Use replacement notation)

(c) [2 marks] What is $\mathbb{Z}/\sim$? (Use set-roster notation)

Answer: (a) $\{4k : k \in \mathbb{Z}\}$, **(b)** $\{4k + 1 : k \in \mathbb{Z}\}$ **(c)** $\{[0], [1], [2], [3]\}$ **Marking scheme: 1 mark for answer. For part(a) and (b), 1 mark for the correct notation. For part(c) -> 1 mark only if 4 distinct sets are shown.**

17. [4 marks] Let $A = \{a, b, c, d\}$. Let $\mathcal{R}$ be a set of reflexive and anti-symmetric relations on A. Let's define a partial order relation $\preccurlyeq$ on $\mathcal{R}$ as follows:

$$\forall R_1, R_2 \in \mathcal{R}\ (R_1 \preccurlyeq R_2 \leftrightarrow R_1 \subseteq R_2)$$

(a) [2 marks] What is the smallest element of $\mathcal{R}$? You must write NA if the element does not exist.

(b) [2 marks] What is the largest element of $\mathcal{R}$? You must write NA if the element does not exist.

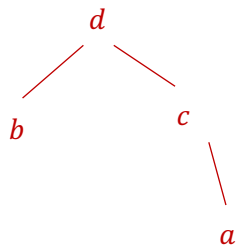
Answer: (a) $\{(a, a), (b, b), (c, c), (d, d)\}$ **(b) NA.**

18. [2 marks] A partial order $\preccurlyeq$ on $A = \{a, b, c, d\}$ has exactly three possible linearisations as follows:

$$a \preccurlyeq^* b \preccurlyeq^* c \preccurlyeq^* d, \qquad a \preccurlyeq^* c \preccurlyeq^* b \preccurlyeq^* d, \qquad b \preccurlyeq^* a \preccurlyeq^* c \preccurlyeq^* d$$

Draw the Hasse Diagram for the poset.

Answer:



Marking scheme: 1 mark for a-c-d. 1 mark for b-d.

No marks for non-trees. No marks for any tree with level more than 4.

=== END OF PAPER ==

General Statistics

Number of students appeared for the exam: 832

Mean: 31.78, Median: 32

Common Errors

MCQs.

- Q6 is incorrectly answered by 533 students. Most common answer is D. Refer to the answers above and it will be clear why statement (iii) is not correct.
- Q4 is incorrectly answered by 449 students. Most common incorrect answer is B followed by D.
- Q12 is incorrectly answered by 441 students. Most common incorrect answer is D. R_1 is not a partial order since it is not an anti-symmetric relation.
- Q10 had two possible interpretation: NOT(true for A, B, C) and (Not true) for A, B, C. Due to these two interpretations, we have accepted both B and E as answers.

Q13A. Errors are either due to not understanding the operators or careless mistakes.

Q13B.

- For the first part, there were a significant portion of people who didn't use Q13A. For the sake of leniency, we accept any other justification such as "Negation of Implication" or "De Morgan's and implication law".
- Many missed justifications such as using specialisation when they meant generalisation or vice versa.
- There were quite a few missed statements made that did not line up with the pre-filled justifications. Such as creating implication statements when the statement needed to be specialised into one of the two terms.

Q14.

- Either missing $\{1\}$ or 2 as an element of A.
- Having $\{\{\{1\}, 2\}\}$ as an element of A
- Having Ø or $\{\emptyset\}$ as an extra element of A.

Q15.

Most common mistake is to not understand it is a relation between pairs. Many students wrote things like $\{(2,2),(3,3),(3,4),\dots\}$, which is denotes a relation between numbers.

Q16A/B.

- Notation mistakes that result in wrong expressions, for example $\{(4\mid k) == 1 \mid k \in \mathbb{Z}\}, \{4k = 7x - 3y \mid k, x, y \in \mathbb{Z}\})$
- Simplifying almost correctly, but not excluding non-integers: for example, $\{(7x - 3y)\ /\ 4 \mid x, y \in Z\}$
- 16a was correctly identified as multiples of 4, 16b answer was multiples of -3

Q16C.

- Less than 4 equivalence classes, say $\{[0],[1],[3]\}$
- Identifying 4 equivalence classes but not using representatives from different classes, for example $\{[-1],[0],[3],[4]\}$

Q17.

- Missing the point of a reflexive relation; and that the smallest element exists
- Failing to recognize that there are multiple maximal elements.

Q18

Writing a hierarchical tree structure where d is on top of all elements and those elements are not related to each other.