NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

MID-TERM TEST
AY2026/27 Semester 1

CS1231S — DISCRETE STRUCTURES

7 October 2026 Time Allowed: 1 hour 30 minutes

INSTRUCTIONS

1. This assessment paper contains **EIGHTEEN (18)** questions in **TWO (2)** parts and comprises **SIX (6)** printed pages.
2. This is an **OPEN BOOK** assessment.
3. Printed/written materials are allowed. Apart from calculators, electronic devices are not allowed.
4. Answer **ALL** questions on the **ANSWER SHEET** provided.
5. Shade your Student Number and MCQ answers **ONLY with a pencil** (2B or above).
6. You may write your answers for Part B in in pen or pencil, though we advise you to use pencil in case you need to erase and rewrite your answers. Write legibly, or marks may be deducted.
7. You are to write within the boxes provided in the Answer Sheet. No extra pages should be submitted as it will jam the scanning machine.
8. The maximum mark of this assessment is 50.
9. Do not start writing or flip over this page until you are told to do so.

——— END OF INSTRUCTIONS ———

Part A: Multiple Choice Questions (Total: 24 marks)

Each multiple choice question (MCQ) is worth <u>two marks</u> and has exactly **one** correct answer.

1. Consider the following statements.

   (i) $\sim (p \leftrightarrow q) \wedge \sim (q \leftrightarrow r) \wedge \sim (r \leftrightarrow p)$
   (ii) $(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$
   (iii) $(p \wedge q) \vee (q \wedge r) \vee (r \wedge p)$
   (iv) $(p \vee q \vee r) \wedge (\sim p \wedge \sim q \wedge \sim r)$
   (v) $(p \wedge \sim p) \vee (q \wedge \sim q) \vee (r \wedge \sim r)$

   What is the largest number of statements from above that are equivalent to one another?

   A. 1.
   B. 2.
   C. 3.
   D. 4.
   E. 5.

2. Let $B = \{true, false\}$. Consider the following sets and their cardinalities.

   (i) $a = |\{(p, q) \in B \times B : p \wedge q \equiv true\}|$
   (ii) $b = |\{(p, q) \in B \times B : p \vee q \equiv true\}|$
   (iii) $c = |\{(p, q) \in B \times B : p \leftrightarrow q \equiv true\}|$

   Which of the following statement is correct for the set $N = \{a, b, c\}$?

   A. $\exists n \in N\ n = 0$.
   B. $\forall n \in N\ \exists m \in N\ m = n + 1$.
   C. $\exists m \in N\ \forall n \in N\ m = n + 1$.
   D. $\exists!\, n \in N\ Even(n)$.
   E. None of options (A), (B), (C), (D) are correct.

3. Which of the following is/are always true for any finite sets A and B?
   (Note: $\mathcal{P}(X)$ is the power set of X.)

   (i) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$
   (ii) $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$
   (iii) $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$
   (iv) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

   A. Only (ii).
   B. Only (iv).
   C. Only (i) and (ii).
   D. Only (iii) and (iv).
   E. None of options (A), (B), (C), (D) are correct.

4. Let $A = \{A_1, A_2, A_3, \cdots, A_m\}$ and $B = \{B_1, B_2, B_3, \cdots, B_n\}$ be partitions of a non-empty set S. Consider

$$C = \{A_i \cap B_j : (1 \leq i \leq m) \wedge (1 \leq j \leq n) \wedge (A_i \cap B_j \neq \emptyset)\}.$$

Which of the following statements is correct?

A. C is always a partition of S.
B. C is a partition of S only if $A = B$.
C. C is a partition of S only if $m = n$.
D. C needs not be a partition of S because two intersections may overlap.
E. None of options (A), (B), (C), (D) are correct.

5. Suppose R and S are equivalence relations on a nonempty set A. Which condition is sufficient to guarantee that $R \cup S$ is also an equivalence relation on A?

A. $R \cap S \neq \emptyset$
B. $R \subseteq S$ or $S \subseteq R$
C. $R \neq S$
D. There is no way for $R \cup S$ to be an equivalence relation on A.
E. None of options (A), (B), (C), (D) are correct.

6. Which of the following statements is/are always true for all finite sets A and B?

(i) $A \times B = B \times A$
(ii) $|A \times B| = |A| \times |B|$
(iii) If $A \neq B$, then $A \times B \neq B \times A$.
(iv) If $A \times B = \emptyset$, then $(A = \emptyset) \vee (B = \emptyset)$.

A. Only (iv).
B. Only (ii) and (iv).
C. Only (i), (ii) and (iv).
D. Only (ii), (iii) and (iv).
E. None of options (A), (B), (C), (D) are correct.

7. Suppose R is an equivalence relation on a nonempty set A, and $a, b \in A$.
Which of the following statements is/are not necessarily true?

(i) $a\ R\ b \rightarrow [a] = [b]$
(ii) $[a] \cap [b] \neq \emptyset \rightarrow [a] = [b]$
(iii) $a \neq b \rightarrow [a] \neq [b]$

A. Only (i).
B. Only (iii).
C. Only (i) and (ii).
D. Only (ii) and (iii).
E. None of options (A), (B), (C), (D) are correct.

8. For each integer $n \geq 2$, define

$$A_n = \{k \in \mathbb{Z}^+ : n \mid k\} \setminus \{n\}.$$

Which of the following is correct?

(Note: Composite number is defined in Lecture 4: A natural number n is composite iff $n = rs$ for some integers r and s with $1 < r < n$ and $1 < s < n$.)

A. $\bigcup_{n=2}^{\infty} A_n = \mathbb{Z}^+$

B. $\bigcup_{n=2}^{\infty} A_n = \mathbb{Z}^+ \setminus \{1\}$

C. $\bigcup_{n=2}^{\infty} A_n = \mathbb{Z}^+ \setminus \{1,2\}$

D. $\bigcup_{n=2}^{\infty} A_n$ is the set of composite integers

E. None of options (A), (B), (C), (D) are correct.

9. Let R be a relation on a nonempty set A. Suppose R is both symmetric and antisymmetric. What can be concluded?

A. R must be empty.

B. R must be the identity relation $\{(a, a) : a \in A\}$.

C. $\forall x, y \in A \left((x, y) \in R \rightarrow x = y\right)$.

D. R must be reflexive.

E. None of options (A), (B), (C), (D) are correct.

10. Which of the following identities is/are NOT true for all sets A, B, C?

(i) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

(ii) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

(iii) $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$

(iv) $A \cup (B \setminus C) = (A \cup B) \setminus (A \cup C)$

A. Only (i).

B. Only (iv).

C. Only (i) and (ii).

D. Only (iii) and (iv).

E. None of options (A), (B), (C), (D) are correct.

11. Consider a relation $R = \{(1, 1), (2, 3)\}$ over the domain $\{1, 2, 3\}$. Which of the following options is correct?

A. R is an equivalence relation.

B. $R \circ R$ is an equivalence relation.

C. $R \circ R^{-1}$ is an equivalence relation.

D. $R^{-1} \circ R$ is an equivalence relation.

E. None of options (A), (B), (C), (D) are correct.

12. Consider the following relations defined on $\mathbb{N} \times \mathbb{N}$:

$$\forall (a,b),(c,d) \in \mathbb{N} \times \mathbb{N}\ \big((a,b)R_1(c,d) \leftrightarrow a \leq c\big)$$

$$\forall (a,b),(c,d) \in \mathbb{N} \times \mathbb{N}\ \big((a,b)R_2(c,d) \leftrightarrow (a \leq c) \wedge (b \geq d)\big)$$

Which of the following options is correct about these relations?

A. Both R_1 and R_2 are not partial order relations.
B. R_1 is a partial order relation but R_2 is not a partial order relation.
C. R_1 is not a partial order relation but R_2 is a partial order relation.
D. Both R_1 and R_2 are partial order relations.
E. Either R_1 or R_2 is a total order relation.

Part B: Short Answer Questions (Total: 26 marks)

13. (a) [1 mark] Rewrite $\sim(\sim p \rightarrow q)$ using only negation and conjunction. Only write the final answer.

(b) [4 marks] Consider a proof by contradiction for the following argument:

$$\begin{array}{ll} p \rightarrow q & (Premise\ P1) \\ \sim r \vee s & (Premise\ P2) \\ p \vee r & (Premise\ P3) \\ \hline \sim q \rightarrow s & (Conclusion) \end{array}$$

Fill in the missing blanks below in the Answer Sheet to complete the proof.

	Statement	Justification
1	$\sim(\sim q \rightarrow s)$	Assumption
2		
3	$\sim s$	Specialisation on 2
4	$\sim r \vee s$	Premise P2
5	$\sim r$	
6	$p \rightarrow q$	Premise P1
7	$\sim q$	
8		
9	$p \vee r$	
10	r	Elimination on 8, 9
11	$r \wedge \sim r$	
12	$\boldsymbol{false}$	Negation Law on 11
13	$\therefore \sim q \rightarrow s$	Conclusion using proof by Contradiction

14. [3 marks] Write the smallest possible set A in the set-roster notation that satisfies the following conditions: $\{\{1\}, 2\} \subseteq A$ and $\{\{1\}, 2\} \in A$.

15. [6 marks] Let $A = \{2, 3, 6\}$. We define two relations R and S on $A \times A$ as follows:

$$(a, b)R(c, d) \leftrightarrow (a \leq b) \wedge (c \leq d) \wedge (ab = c + d)$$

$$(a, b)S(c, d) \leftrightarrow (a \leq b) \wedge (c \geq d) \wedge (a + b = cd)$$

    (a) [2 marks] Write down R in set-roster notation.

    (b) [4 marks] Write down $(S \circ R)^{-1}$ in set-roster notation.

16. [6 marks] Define an equivalence relation $\sim$ on $\mathbb{Z}$ such that $x \sim y \leftrightarrow 4 \mid (7x - 3y)$.

    (a) [2 marks] What is $[4]$? (Use replacement notation)

    (b) [2 marks] What is $[-3]$? (Use replacement notation)

    (c) [2 marks] What is $\mathbb{Z}/\sim$? (Use set-roster notation)

17. [4 marks] Let $A = \{a, b, c, d\}$. Let $\mathcal{R}$ be a set of reflexive and anti-symmetric relations on A. Let's define a partial order relation $\preccurlyeq$ on $\mathcal{R}$ as follows:

$$\forall R_1, R_2 \in \mathcal{R}\ (R_1 \preccurlyeq R_2 \leftrightarrow R_1 \subseteq R_2)$$

    (a) [2 marks] What is the smallest element of $\mathcal{R}$? You must write NA if the element does not exist.

    (b) [2 marks] What is the largest element of $\mathcal{R}$? You must write NA if the element does not exist.

18. [2 marks] A partial order $\preccurlyeq$ on $A = \{a, b, c, d\}$ has exactly three possible linearisations as follows:

$$a \preccurlyeq^* b \preccurlyeq^* c \preccurlyeq^* d, \qquad a \preccurlyeq^* c \preccurlyeq^* b \preccurlyeq^* d, \qquad b \preccurlyeq^* a \preccurlyeq^* c \preccurlyeq^* d$$

    Draw the Hasse Diagram for the poset.

=== END OF PAPER ==