

**CS1231S: Discrete Structures**  
**Tutorial #3: Sets**  
**(Week 5: 8– 12 September 2025)**

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### 1. Discussion Questions

These are meant for your own discussion. No answers will be provided.

D1 Which of the following are true? Which are false?

- |   |  |                                      |
|---|--|--------------------------------------|
| (a) $\emptyset \in \emptyset$ .           | (e) $\{\emptyset, 1\} = \{1\}$ .           | (i) $\emptyset \in \{1, 2\}$ .       |
| (b) $\emptyset \subseteq \emptyset$ .     | (f) $1 \in \{\{1, 2\}, \{2, 3\}, 4\}$ .    | (j) $\{1, 2, 3\} = \{3, 1, 1, 2\}$ . |
| (c) $\emptyset \in \{\emptyset\}$ .       | (g) $\{1, 2\} \subseteq \{3, 2, 1\}$ .     | (k) $\{5\} \in \{2, 5\}$ .           |
| (d) $\emptyset \subseteq \{\emptyset\}$ . | (h) $\{3, 3, 2\} \subsetneq \{3, 2, 1\}$ . | (l) $\{1, 2\} \in \{1, \{2, 1\}\}$ . |

D2. Let  $A = \{0, 1, 4, 5, 6, 9\}$  and  $B = \{0, 2, 4, 6, 8\}$ . Find  $|A \cap B|$  and  $|A \cup B|$ .

D3. Let  $A = \{1, \{1, 2\}, 2, \{2, 1, 1\}\}$ . Find  $|A|$  and write out  $A \times A$  in its simplest form (that is, with no duplicate elements).

### 2. Tutorial Questions

Note that the sets here are finite sets, unless otherwise stated.

1. Let  $\mathcal{P}(A)$  denotes the power set of  $A$ . Find the following:

- $\mathcal{P}(\{a, b, c\})$ ;
- $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$ .

2. Let the universal set be  $\mathbb{R}$ ,  $A = \{x \in \mathbb{R} : -2 \leq x \leq 1\}$  and  $B = \{x \in \mathbb{R} : -1 < x < 3\}$ .

Note that  $A$  and  $B$  may be written as  $[-2, 1]$  and  $(-1, 3)$  respectively using real numbers interval notation (Lecture #5 slide 27). Find the following and write your answers in set-builder notation, and if appropriate, also in interval notation.

- $A \cup B$ ;
- $A \cap B$ ;
- $\bar{A}$ ;
- $\bar{A} \cap \bar{B}$ ;
- $A \setminus B$ .

3. (AY2022/23 Sem2 mid-term test.)

For each of the following statements, prove whether it is true or false.

- There exist non-empty finite sets  $A$  and  $B$  such that  $|A \cup B| = |A| + |B|$ .
- There exist non-empty finite sets  $A$  and  $B$  such that  $|A \cup B| \neq |A| + |B|$ .
- There exist finite sets  $A$  and  $B$  such that  $A \times \mathcal{P}(B) = \mathcal{P}(A \times B)$ .

Aiken thought that (c) is false and he provided a proof as follows:

1. Let  $|A| = n, |B| = k$ .
2. Then  $|\mathcal{P}(B)| = 2^k$  (cardinality of power set).
3.  $|A \times B| = nk$  (cardinality of Cartesian product) and hence  $|\mathcal{P}(A \times B)| = 2^{nk}$  (cardinality of power set).
4. Then  $|A \times \mathcal{P}(B)| = n2^k$  (cardinality of Cartesian product).
5. Since  $|A \times \mathcal{P}(B)| = n2^k \neq 2^{nk} = |\mathcal{P}(A \times B)|$ , therefore the statement is false.

Is Aiken's conclusion that the statement is false correct? Is his proof correct? If his proof is not correct, can you write a correct proof?

4. Let  $A = \{2n + 1 : n \in \mathbb{Z}\}$  and  $B = \{2n - 5 : n \in \mathbb{Z}\}$ . Is  $A = B$ ? Prove that your answer is correct. What does this tell us about how odd numbers may be defined?

5. Using definitions of set operations (also called the **element method**), prove that for all sets  $A, B, C$ ,

$$A \cap (B \setminus C) = (A \cap B) \setminus C.$$

6. (Past year's midterm test question.)  
Using **set identities** (Theorem 6.2.2), prove that for all sets  $A, B$  and  $C$ ,

$$A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C).$$

7. For sets  $A$  and  $B$ , define  $A \oplus B = (A \setminus B) \cup (B \setminus A)$ .
  - a. Let  $A = \{1,4,9,16\}$  and  $B = \{2,4,6,8,10,12,14,16\}$ . Find  $A \oplus B$ .
  - b. Using **set identities** (Theorem 6.2.2), prove that for all sets  $A$  and  $B$ ,

$$A \oplus B = (A \cup B) \setminus (A \cap B).$$

8. Let  $A$  and  $B$  be set. Show that  $A \subseteq B$  if and only if  $A \cup B = B$ .

9. Consider the claim:

$$\text{For all sets } A, B, C, \quad (A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B).$$

Aiken wrote the following proof:

1.  $x \in (A \setminus B) \cup (B \setminus A)$
2.  $\Rightarrow x \in (A \setminus B)$  or  $x \in (B \setminus A)$
3.  $\Rightarrow x \in A$  and  $x \notin B$  or  $x \in B$  and  $x \notin A$
4.  $\Rightarrow x \in A$  or  $x \in B$  and  $x \notin A$  and  $x \notin B$
5.  $\Rightarrow x \in (A \cup B)$  and  $x \in \overline{(A \cap B)}$
6.  $\Rightarrow x \in (A \cup B) \cap \overline{(A \cap B)}$
7.  $\Rightarrow x \in (A \cup B) \setminus (A \cap B)$ .

$$\text{Therefore, } (A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B).$$

- (a) What is wrong with Aiken's proof?
- (b) Prove or disprove the claim.

10. Hogwarts School of Witchcraft and Wizardry where Harry Potter attended was divided into 4 houses: Gryffindor, Hufflepuff, Ravenclaw and Slytherin.

Let  $HSWW$  be the set of students in the Hogwarts School of Witchcraft and Wizardry, and  $G, H, R$  and  $S$  be the sets of students in the 4 houses.

What are the necessary conditions for  $\{G, H, R, S\}$  to be a partition of  $HSWW$ ? Explain in English and the write logical statements.



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For questions 11 and 12, for sets  $A_m, A_{m+1}, \dots, A_n$ , we define the following:

$$\bigcup_{i=m}^n A_i = A_m \cup A_{m+1} \cup \dots \cup A_n$$

and

$$\bigcap_{i=m}^n A_i = A_m \cap A_{m+1} \cap \dots \cap A_n$$

11. Let  $A_i = \{x \in \mathbb{Z} : i \leq x \leq 2i\}$  for all integers  $i$ . Write  $A_{-2}$ ,  $\bigcup_{i=3}^5 A_i$  and  $\bigcap_{i=3}^5 A_i$  in set-roster notation, set-builder notation, and interval notation.
12. Let  $V_i = \left\{x \in \mathbb{R} : -\frac{1}{i} \leq x \leq \frac{1}{i}\right\} = \left[-\frac{1}{i}, \frac{1}{i}\right]$  for all positive integers  $i$ .
- What is  $\bigcup_{i=1}^4 V_i$ ?
  - What is  $\bigcap_{i=1}^4 V_i$ ?
  - What is  $\bigcup_{i=1}^n V_i$ , where  $n$  is a positive integer?
  - What is  $\bigcap_{i=1}^n V_i$ , where  $n$  is a positive integer?
  - Are  $V_1, V_2, V_3, \dots$  mutually disjoint?