

CS2100: Computer Organisation
Tutorial #6: Boolean Algebra, Logic Gates and Simplification
(Week 8: 9 – 13 March 2026)

Self-Exploratory Questions

SE1. (a) One common mistake that students make is the following: $A \cdot B + A' \cdot B' = 1$... (equation 1)

This seems to be erroneously “derived” from the following rule: $X + X' = 1$.
Explain why the rule is wrongly applied here.

(b) Is the following equation correct? Why? $A \cdot B + (A \cdot B)' = 1$... (equation 2)

SE2. Given the following two 3-variable Boolean functions:

$$F(A,B,C) = \sum m(0, 2, 4, 6, 7)$$

$$G(A,B,C) = \sum m(1, 2, 3, 6)$$

- (a) Write the product-of-maxterms expressions in ΠM notation for F and G .
- (b) If $X = F + G$, write the sum-of-minterms expressions in $\sum m$ notation for X .
- (c) If $Y = F \cdot G$, write the sum-of-minterms expressions in $\sum m$ notation for Y .
- (d) If $Z = F \oplus G$, write the sum-of-minterms expressions in $\sum m$ notation for Z .

Do you know how to generalise the above for any arbitrary Boolean functions F and G ?

[If it is not convenient to type symbols like \sum and Π in the forums on Canvas or QnA, you may use Sum-m to mean $\sum m$ and Prod-M to mean ΠM . Example: Sum-m(0, 2, 4, 6, 7), Prod-M(2, 3, 5).]

SE3. For each part below, how many prime implicants (PIs) and essential prime implicants (EPIs) are there in the K-map? What is/are the simplified **SOP expressions**? List out all alternative answers. [$d(\dots)$ and $D(\dots)$ denote don't-cares.]

(a) $F1(A,B,C,D) = \sum m(5, 8, 10, 12, 13, 14)$

(b) $F2(W,X,Y,Z) = \Pi M(0, 1, 2, 8, 9, 10)$

(c) $F3(K,L,M,N) = \sum m(1, 7, 10, 13, 14) + d(0, 5, 8, 15)$

(In general, students are weak at identifying the EPIs (essential prime implicants) correctly. Find out how Quine-McCluskey's method works; though not in the syllabus, it may help you understand better.)

SE4. For each of the functions in D3 above, find the simplified **POS expression**. List out all alternative answers, if any.

You are encouraged to do the above self-exploratory questions on your own, and discuss them on Canvas or QnA if you like to. These are fundamental concepts that you must know before you attempt the tutorial questions below.

Important notes:

This is an important tutorial to get the basics right. K-maps are used extensively in logic design. A common weakness among students is the determination of prime implicants and essential prime implicants. If you still have doubts after the tutorial, try go through the slides on “Quine McCluskey technique” (non-examinable) as that explains the underlying working of K-maps.

By default, we assume that complemented literals are NOT available, unless otherwise stated. Logic constants (0 and 1) are always available, and they are considered (degenerate form of) SOP and POS expressions. The above are to be assumed from now onwards and may not be repeated in future tutorials, assignments and final exam.

Do not omit the \cdot operator when writing product terms such as $A \cdot B$. Omitting the \cdot operator, like AB , means that AB is a 2-bit value. Marks will be deducted.

Tutorial questions

1. Using Boolean algebra, simplify each of the following expressions into simplified **sum-of-products (SOP) expression**. Indicate the law/theorem used at every step.

(a) $F(x,y,z) = (x + y \cdot z') \cdot (y' + y) + x' \cdot (y \cdot z' + y)$

(b) $G(p,q,r,s) = \prod M(5, 9, 13)$

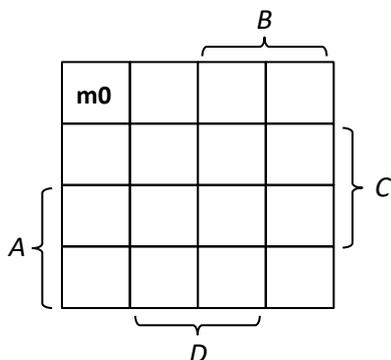
[Tip: For (b), it is easier to start with the given expression and get done in about 6 steps, rather than expanding it into sum-of-products/sum-of-minterms expression first.]

2. We assume you have done the self-exploratory questions above.

Present your answers for the expressions in (a), (b), (c) in SE3. You do not need to draw the K-maps. Now, draw the K-map for (d) below, determine the number of PIs and EPIS in the K-map, and find the simplified SOP and POS expressions.

(d) $F4(A,B,C,D) = \prod M(4, 8, 9, 11, 12) \cdot D(2, 3, 6, 7, 10, 14)$

3. (a) The following K-map layout is used for a 4-variable Boolean function $T(A,B,C,D)$. Fill in the minterm positions m1 to m15 into the respective cells. m0 has been filled for you.



(b) Given the following 4-variable Boolean function:

$$T(A,B,C,D) = \prod M(3,7,8,10,12,13) \cdot X(6,11,14,15)$$

where X 's are the don't-cares, write out the Σm notation for $T(A,B,C,D)$.

- (c) Draw the K-map for T using the layout above.
- (d) How many PIs (prime implicants) are there in the K-map? List out all the PIs.
- (e) How many EPIs (essential prime implicants) are there? List out all the EPIs.
- (f) What is the simplified SOP expression for T ? List out all alternative solutions.
- (g) What is the simplified POS expression for T ? List out all alternative solutions.
- (h) Implement the simplified SOP expression for T using a 2-level AND-OR circuit and a 2-level NAND only circuit.

4. A circuit takes in four inputs K,L,M,N and generates 3 outputs X,Y,Z as follow:

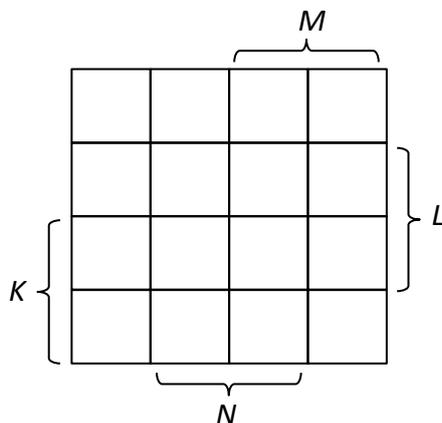
$X(K,L,M,N) = 1$ if $KL = MN$, or 0 otherwise,
where KL and MN are 2-bit unsigned integers.

$Y(K,L,M,N) = 1$ if $KL \leq MN$, or 0 otherwise,
where KL and MN are 2-bit unsigned integers.

$Z(K,L,M,N) = 1$ if $KLM < LMN$, or 0 otherwise,
where KLM and LMN are 3-bit unsigned integers.

For parts (a) – (c) below, you may assume that the input 0000 will not occur.

- (a) Fill in the truth table for the circuit. Write 'd' for don't cares.
- (b) Fill in the K-maps of X , Y and Z using the layout given below.



- (c) Write out the simplified SOP expressions of X , Y and Z .
- (d) After designing the circuit according to the simplified SOP expressions in (c), if you feed the input 0000 into it, what will be the outputs?

(This is an important question that reminds students on the use of don't-cares.)