Programming Language Concepts, CS2104
Lecture 6

Tupled Recursion and Exceptions

Reminder of Last

- Computing with procedures
  - lexical scoping
  - closures
  - procedures as values
  - procedure call
- Higher-Order Programming
  - proc. abstraction
  - lazy arguments
  - genericity
  - loop abstraction
  - folding

Outline

- Recursion vs Iteration (self-reading)
- Tupled Recursion
- Exceptions

Tupled Recursion

Functions with multiple results

Computing Average

```
fun {SumList Ls}
    case Ls  of nil then 0
    [] X|Xs then X+{SumList Xs} end
End

fun {Length Ls}
    case Ls  of nil then 0
    [] X|Xs then 1+{Length Xs} end
end

fun {Average Ls} {Sum Ls}/{Length Ls} end
```

- What is the Problem?

Problem?

- Traverse the same list multiple traversals.
- Solution : compute multiple results in a single traversal!

Tupling - Computing Two Results

```
fun {CPair Ls}
   {Sum Ls}#{Length Ls}
end
```

```
fun {CPair Ls}
    case Ls  of nil then 0#0
    [] X|Xs then  case {CPair Xs}
                  of S#L then (X+S)#(1+L) end
    end
end
```


Using Tupled Recursion

```
fun {Average Ls}
     {Sum Ls}/{Length Ls}
end
```

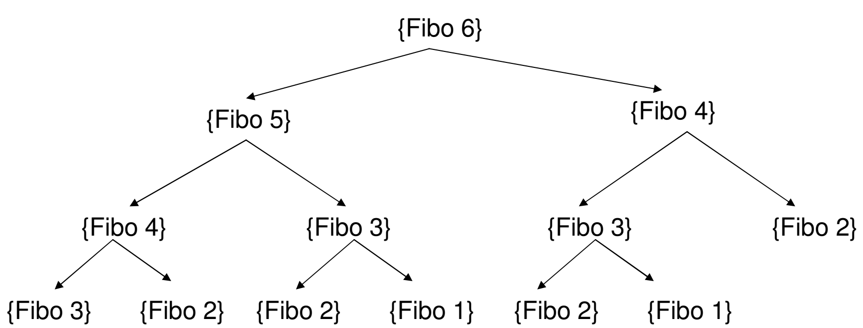
```
fun {Average Ls}
     case {CPair Ls} of S#L then S/L end
end
```

Inefficient Fibonacci

- Time complexity of `{Fibo N}` is proportional to 2^N.

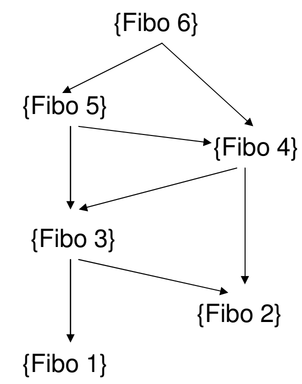
```
fun {Fibo N}
   case N of
      1 then 1
   [] 2 then 1
   [] M then {Fibo (M-1)} + {Fibo (M-2)}
   end
end
```


A Call Tree of Fibo



Many repeated calls!

A Call Graph of Fibo

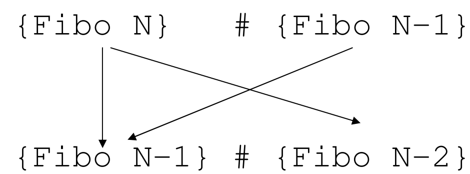


No repeated call through reuse of identical calls

Tupling - Computing Two Results

```
fun {FPair N}
   {Fibo N}#{Fibo N-1}
end
```

Compute two calls from next two:



Tupling - Computing Two Results

```
fun {FPair N}
   {Fibo N}#{Fibo N-1}
end
```

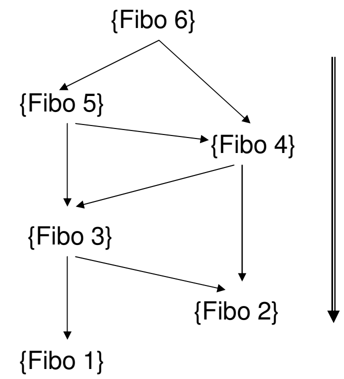
```
fun {FPair N}
    case N of
       2 then 1#1
    [] M then  case {FPair M-1}
               of S#L then (S+L)#S end
    end
end
```


Using the Tupled Recursion

```
fun {Fibo N}
 case {Fibo N+1}#{Fibo N} of
        A#B then B end
end
```

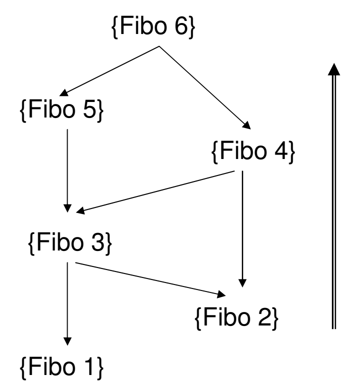
```
fun {Fibo N}
  case {FPair N+1} of A#B then B end
end
```


Linear Recursion




```
fun {FPair N}
    case N  of 2 then 1#1
    [] M then  case {FPair M-1}
      of S#L then (S+L)#S end
     end
end
```


To Iteration




```
{FPair N} = {H^(N-2) 1#1}
          = {FPairIt (N-2) 1#1}
fun {H P}
  case P of A#B then  A+B#A end
end
```

Tail-Recursive Fibonacci

```
fun {FPair N}  {FPairIt (N-2) 1#1} end

fun {FPairIt N P}
  case N of
       0 then P
    [] M then {FPairIt N-1 {H P}} end
end
```


Summary So Far

- Tupled Recursion
  - Eliminate multiple traversals
  - Eliminate redundant calls
- Eureka – find suitable tuple of calls.

Exceptions

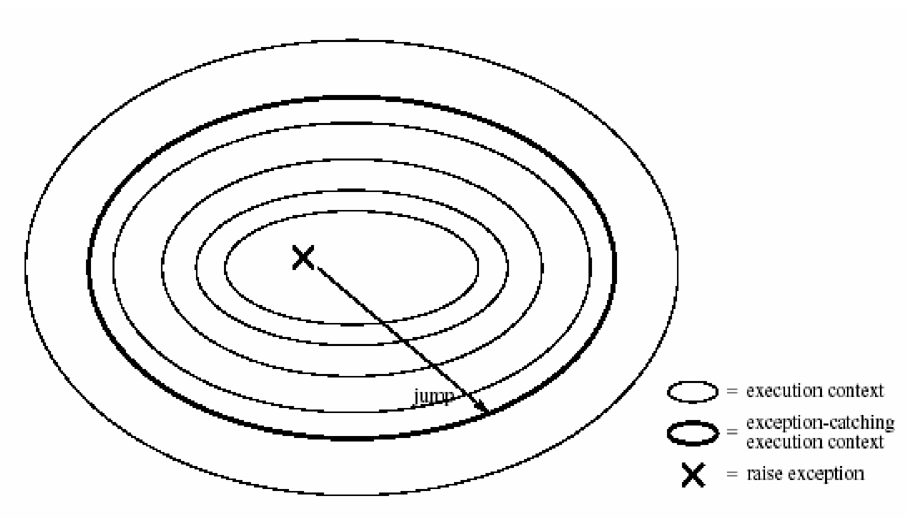
Exceptions

- Error = Actual behavior - Desired behavior.
- Type of errors:
  - Internal: invoking an operation with an illegal type/value
  - External: opening a nonexisting file
- Detect and handle these errors without stopping the program execution.
- Solution - Transfer to an **exception handler**, and pass a value that describes the error.

Exceptions handling

- Oz program = interacting "**components**"
- Exception causes a "**jump**" from inside the component to its boundary.
- Able to exit arbitrarily levels of nested contexts.
- A **context** is an entry on the semantic stack.
- Nested contexts are created by procedure calls and sequential compositions.

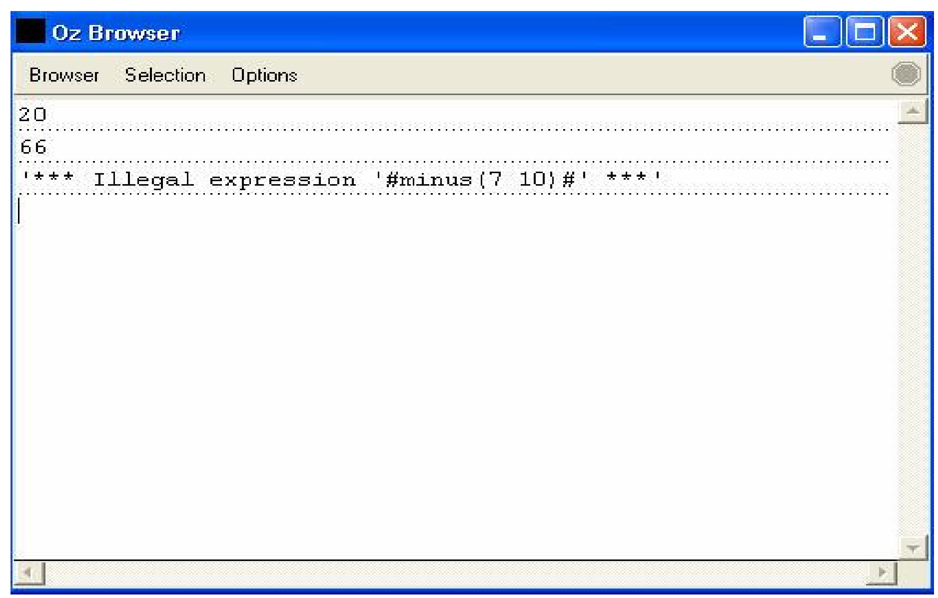
Exceptions handling



Exceptions (Example)

```
fun {Eval E}
  if {IsNumber E} then E
  else
    case E
    of plus(X Y) then {Eval X}+{Eval Y}
    [] times(X Y) then {Eval X}*{Eval Y}
    else raise illFormedExpression(E) end
    end
  end
end
try
  {Browse {Eval plus(plus(5 5) 10)}}
  {Browse {Eval times(6 11)}}
  {Browse {Eval minus(7 10)}}
catch illFormedExpression(E) then
  {Browse '*** Illegal expression '#E#' ***'}
end
```


Exceptions (Example)



Exceptions. `try` and `raise`

- **`try`:** creates an exception-catching context together with an exception handler.
- **`raise`:** jumps to the boundary of the innermost exception-catching context and invokes the exception handler there.
- **`try`** <S> **`catch`** <X> **`then`** <S>$_1$ **`end`:**
  - if <S> does not raise an exception, then execute <s>.
  - if <S> raises an exception, then the (still ongoing) execution of <S> is aborted. All information related to <S> is popped from the semantic stack. Control is transferred to <S>$_1$, passing it a reference to the exception in <X>.

Exceptions. Full Syntax

- A **`try`** statement can specify a **`finally`** clause which is always executed, whether or not the statement raises an exception.
- **`try`** <S>$_1$ **`finally`** <S>$_2$ **`end`**
  is equivalent to:
- **`try`** <S>$_1$

```
catch X then
    <S>2
    raise X end
end
<S>2
```

where an identifier `X` is chosen that is not free in <s>$_2$

Exceptions. Full Syntax (Example)

- An example with `catch` and `finally`.
- **`try`**

```
{ProcessFile F}
catch X then
{Browse '*** Exception '#X#
   ' when processing file ***'}
finally {CloseFile F} end
```

- Similar with two nested **`try`** statements!

System Exceptions

- Raised by Mozart system
- `failure`: attempt to perform an inconsistent bind operation in store (“unification failure”);
- `error`: run-time error inside a program, like type or domain errors;
- `system`: run-time condition in the environment of the Mozart, like failure to open a connection between two processes.

System Exceptions (Example)

```
functor
import
  Browser
define
  fun {One} 1 end
  fun {Two} 2 end
  try
    {One}={Two}
  catch
    failure(...) then
      {Browser.browse 'We caught the failure'}
  end
end
```


Summary

- Recursion vs Iteration
- Tupled Recursion
- Exceptions

Reading suggestions

- Chapter 2, Sections 2.4, 2.5, 2.6, 2.7 from [van Roy,Haridi; 2004]
- Exercises 2.9.4-2.9.12 from [van Roy,Haridi; 2004]

Thank you for your attention!

Reverse

- Reversing a list
- How to reverse the elements of a list

```
{Reverse [a b c d]}
```

returns

```
[d c b a]
```


Reversing a List

- Reverse of `nil` is `nil`
- Reverse of `X|Xr` is `Z`, where
  reverse of `Xr` is `Yr`, and
  append `Yr` and `[X]` to get `Z`

```
{Rev [a b c d]}=                         =[d c b a]
{Rev a|[b c d]}={Append {Rev [b c d]} [a]}=[d c b a]
{Rev b|[c d]}={Append {Rev [c d]} [b]}    =[d c b]
{Rev c|[d]}={Append {Rev [d]} [c]}        =[d c]
{Rev d|nil}={Append {Rev nil} [d]}        =[d]
                       nil
```


Question

- What is correct

```
{Append {Reverse Xr} X}
```

or

```
{Append {Reverse Xr} [X]}
```


Naive Reverse Function

```
fun {NRev Xs}
   case Xs of
      nil  then nil
   [] X|Xr then {Append {NRev Xr} [X]}
   end
end
```

Question

- What is the problem with the naive reverse?
- Possible answers
  - not tail recursive
  - `Append` is costly:
    - there are `O{|L1|}` calls

```
fun {Append L1 L2}
   case L1 of
      nil then L2
   [] H|T then H|{Append T L2}
   end
end
```


Cost of Naive Reverse

- Suppose a recursive call `{NRev Xs}`
  - where `{Length Xs}=`n
  - assume cost of `{NRev Xs}` is c(n)

    number of function calls
  - then c(0) = 0

    c(n) = c(`{Append {NRev Xr} [X]}`) + c(n-1)

    = (n-1) + c(n-1)

    = (n-1) + (n-2) + c(n-3) =…= n-1 + (n-2) + … + 1
  - this yields: c(n) = $\frac{n(n-1)}{2}$
- For a list of length n, `NRev` uses approx. n^2 calls!

Doing Better for `Reverse`

- Use an accumulator to capture currently reversed list
- Some abbreviations
  - `{IR Xs}` for `{IterRev Xs}`
  - `Xs ++ Ys` for `{Append Xs Ys}`

Computing `NRev`

```
{NRev [a b c]}                  =
{NRev [b c]}++[a]               =
({NRev [c]}++[b])++[a]          =
(({NRev nil}++[c])++[b])++[a]   =
((nil++[c])++[b])++[a]          =
([c]++[b])++[a]                 =
[c b]++[a]                      =
[c b a]
```

Computing `IterRev (IR)`

```
{IR [a b c] nil}         =
{IR [b c]   a|nil }      =
{IR [c]     b|a|nil}     =
{IR nil     c|b|a|nil}   =
[c b a]
```

- The general pattern:

```
{IR X|Xr Rs} ⇒ {IR Xr X|Rs}
```


Why is Iteration Possible?

Associative Property

```
{Append {Append RL [a]} [b]}
          = {Append RL {Append [a] [b]}}
```

More Generally

```
{Append {Append RL [a]} Acc}
          = {Append RL {Append [a] Acc}}
          = {Append RL  a|Acc }
```


`IterRev` Intermediate Step

```
fun {IterRev Xs Ys}
   case Xs of
      nil  then Ys
   [] X|Xr then {IterRev Xr X|Ys}
   end
end
```

- Is tail recursive now

`IterRev` Properly Embedded

```
local
   fun {IterRev Xs Ys}
      case Xs
      of nil  then Ys
      [] X|Xr then {IterRev Xr X|Ys}
      end
   end
in
   fun {Rev Xs} {IterRev Xs nil} end
end
```

State Invariant for `IterRev`

- Unroll the iteration a number of times, we get:

  `{IterRev` [$X_1 \ldots X_n$] `W}`

  =

  `{IterRev` [$X_{i+1} \ldots X_n$] [$X_i \ldots X_1$]`++W}`

Reasoning for `IterRev` and `Rev`

- **Correctness:**

  `{Rev Xs}` is `{IterRev Xs nil}`
- Using the state invariant, we have:

  `{IterRev` [$X_1 \ldots X_n$] `nil}=`

  = `{IterRev nil` [$X_n \ldots X_1$]`}`

  = [$X_n \ldots X_1$]
- Thus: `{Rev` [$X_1 \ldots X_n$]`}=`[$X_n \ldots X_1$]
- **Complexity:**
- The number of calls for `{IterRev L nil}`, where list `L` has `N` elements, is c(`N`)=`N`

Summary So Far

- Use accumulators
  - yields iterative computation
  - find state invariant
- Loop = Tail Recursion and is a special case of general recursion.
- Exploit both kinds of knowledge
  - on how programs execute **(abstract machine)**
  - on application/problem domain