Programming Language Concepts, CS2104 (17th Sep 2007)

Tutorial 4 Lambda Calculus.

Exercise 1. (Free/Bound) Indicate which occurrences of variables are bound and which ones are free in the following expressions.

1. (λ x . z (x (λ x. y(z)))) x
2. (λ a b . c d a b) a b (λ c d . d c) (λ e f . f) e
3. ((λ u v . λ w. w (λ x. x(u)) (v)) (v)) (λ z. λ y. z(y))

Exercise 2. (Substitutions) Perform the following substitutions :

1 [x → λ z . w] (λ y . x)
2 [x → λ z . w] (λ y . x x)
3 [x → λ z . w] (λ y . x ((λ x . x))
4 [x → λ z . w] (λ x . y)
5 [x → λ z . w] (λ w . x)
6 [x → λ z . w] (λ z . x)
7 [x → λ z . w] (λ z . z x)
8 [x → λ x . w] (λ z . z w)

Exercise 3. (Reduction) Reduce the following lambda expressions to their normal form whenever possible.

1 P ≡ (λ x . x (x y)) I where I ≡ λ u . u
2 Y ≡ λ f. Q Q where Q ≡ (λ x . f(x x))
3 L ≡ (λ x. x x y) (λ x. x x y)
4 (λ x. x L) M where M ≡ (λ x . y)

Exercise 4. (Equivalence) Consider the lambda expressions in Q 3. Determine whether the following pairs of expressions are equivalent or not.

1 L and I
2 P and (λ x . x L) M
3 λ a . y and M
4 λ a . y and λ a . z

Exercise 5. (Church boolean) Implement the following two boolean operators in pure lambda calculus.

not - to negate a boolean value
or - find the disjunction of two Boolean values