Question 7 [Algorithm 16 marks].

Let $a[0..n-1]$ be an array of $n$ distinct numbers. If $i < j$ and $a[i] > a[j]$, then the pair $(i, j)$ is called a mismatch of $a$. For example, the array $a[] = [2, 8, 3]$ has one mismatch, $(1, 2)$, whereas $b[] = [1, 2, 3, 4]$ does not have any mismatches.

(a) List all the mismatches of the array $a = [4, 6, 1, 2]$.

(b) How many arrays of size $n$ have more than $n(n + 1)/2$ mismatches?

(c) Devise an algorithm to calculate the number of mismatches in an integer array $a$ of size $n$. You may assume that all array elements are distinct and the first element is stored at position 0. You will be given full marks if your solution runs in $O(n \log n)$. A sketch of the algorithm is sufficient.