Assignment 03:
Propositional Logic II; Predicate Logic I

Submission on A-4 paper (use as many sheets as you want), to the office COM2, 03-51 (a box is provided). Staple or tie your sheets together and write your name and matriculation number on the top of the front page. Latest submission: Thursday, 9/9/08, 11:00am.

1. (4 marks) Compute a formula in CNF that is equivalent to the following formula

\[(\neg p) \to \neg (q \land \neg (r \lor \neg (s \land u)))\]

by applying the algorithm given in the notes. Indicate the result of each step of the algorithm.

2. (8 marks) Prove Lemma 1 in the notes “Propositional Logic Part II”. Remember that “iff” indicates two directions.

3. (7 marks) Let \(P\) be a predicate symbol of arity 0, \(Q\) be a predicate symbol of arity 2, \(f\) be a function symbol of arity 2, and \(g\) a function symbol of arity 1. Consider the formula

\[\phi = \neg(\exists w((P \lor (\forall v Q(f(x,v),w))) \land (\forall z Q(f(w,x), f(v,z))))\]

(a) Draw the parse tree of \(\phi\).

(b) Indicate the free and bound variables in this parse tree.

(c) Compute \([x \Rightarrow g(f(x,x))]\phi\) and \([w \Rightarrow f(z,v)]\phi\).

4. (6 marks) Let \(\phi\) be the sentence

\[\forall x \forall y \exists z(R(x,y) \to R(y,z))\]

where \(R\) is a predicate symbol of arity 2.

(a) Let \(A = \{a,b,c,d\}\) and \(R^A = \{(b,c),(b,b),(b,a)\}\). Does \(\mathcal{M} \models \phi\) hold? Justify your answer.

(b) Let \(A' = \{a,b,c\}\) and \(R^{A'} = \{(b,c),(a,b),(c,b)\}\). Does \(\mathcal{M}' \models \phi\) hold? Justify your answer.

5. (5 marks) Prove that the following entailment does not hold:

\[\exists x (\neg P(x) \land Q(x)) \models \forall x (P(x) \to Q(x))\]