CS3234 Logic and Formal Systems

Assignment 07:
Program Verification

Submission on A-4 paper (use as many sheets as you want), to the office COM2, 03-51 (under the door, if necessary). Staple or tie your sheets together and write your name and matriculation number on the top of the front page. Latest submission: Friday, 5/11, 5:00pm.

A proper proof in the proof calculus annotates every line with the name of the rule applied to derive that line. In your solutions to the following questions, indicate clearly what invariant and variant you are using.

1. (4 marks) Consider the following program `plusabs` in the programming language presented in the lectures:

   ```
   if (b > 0) {
       c = a + b;
   } else {
       c = a - b;
   }
   ```

   Give a proof for the following Hoare triple.

   ```
   ⊢_{par} \{ \top \} plusabs \{ c = a + |b| \}
   ```

2. (6 marks) Consider the following program `square`, written in the programming language presented in the lectures:

   ```
   a = 0;
   b = x;
   while (b > 0) {
       a = a + x;
       b = b - 1;
   }
   ```
Give a proof for the following Hoare triple.

\[ \vdash_{tot} \ [ x > 0 \ \square a = x^2 ] \]

3. (6 marks) Consider the following program \( P \), written in the programming language presented in the lectures:

\[
\begin{align*}
x &= 1; \\
i &= y; \\
\text{while } (i <> 0) \{ \\
 &\quad i = i - 1; \\
 &\quad x = x + x; \\
\}
\end{align*}
\]

Prove the following Hoare triple:

\[ \vdash_{tot} \ [ y \geq 1 \ \land P \ [ x = 2^y ] ] \]

4. (8 marks) Consider the following program \( P \), written in the programming language presented in the lectures.

\[
\begin{align*}
z &= 0; \\
b &= 0; \\
x &= 0; \\
\text{while } (z <> y) \{ \\
 &\quad \text{if } (b = 0) \{ \\
 &\quad \quad b = 1 \\
 &\quad \} \quad \text{else} \{ \\
 &\quad \quad b = 0; \\
 &\quad \quad x = x + 1 \\
 &\quad \}\; \\
 &\quad z = z + 1 \\
\}
\end{align*}
\]

Prove the following Hoare triple:

\[ \vdash_{par} \ [ \exists n (n + n = y) \ \land P \ [ x + x = y ] ] \]