

CS3234 Logic and Formal Systems

Assignment 07: Program Verification

Submission on A-4 paper (use as many sheets as you want), to the office COM2, 03-51 (under the door, if necessary). Staple or tie your sheets together and write your name and matriculation number on the top of the front page.
Latest submission: Friday, 5/11, 5:00pm.

A proper proof in the proof calculus annotates every line with the name of the rule applied to derive that line. In your solutions to the following questions, indicate clearly what *invariant* and *variant* you are using.

1. (4 marks) Consider the following program `plusabs` in the programming language presented in the lectures:

```
if (b > 0) {  
    c = a + b;  
} else {  
    c = a - b;  
}
```

Give a proof for the following Hoare triple.

$$\vdash_{\text{par}} \{ \top \} \text{plusabs} \{ c = a + |b| \}$$

2. (6 marks) Consider the following program `square`, written in the programming language presented in the lectures:

```
a = 0;  
b = x;  
while (b > 0) {  
    a = a + x;  
    b = b - 1;  
}
```

Give a proof for the following Hoare triple.

$$\vdash_{\text{tot}} [x > 0] \text{square} [a = x^2]$$

3. (6 marks) Consider the following program P , written in the programming language presented in the lectures:

```
x = 1;
i = y;
while (i <> 0) {
    i = i - 1;
    x = x + x;
}
```

Prove the following Hoare triple:

$$\vdash_{\text{tot}} [y \geq 1] P [x = 2^y]$$

4. (8 marks) Consider the following program P , written in the programming language presented in the lectures.

```
z = 0;
b = 0;
x = 0;
while (z <> y) {
    if (b = 0) {
        b = 1
    } else {
        b = 0;
        x = x + 1
    };
    z = z + 1
}
```

Prove the following Hoare triple:

$$\vdash_{\text{par}} \{ \exists n(n + n = y) \} P \{ x + x = y \}$$