Model Checking (I)

★ Motivation for verification

★ Computation Tree Logic —syntax and semantics

★ Example: mutual exclusion

★ A model checking algorithm

Verification Methods

Verification methods may be classified according to the following main criteria:

- ***Proof-based vs. model-based*** - if a soundness and completeness theorem holds, than:
  - proof = valid formula = true in all models;
  - model-based = check satisfiability in one model
- ***Degree of automation*** - fully automated, partially automated, or manual
- ***Full- vs. property-verification*** - a single property vs. full behavior
- ***Domain of application*** - hardware or software; sequential or concurrent; reactive or terminating; etc.
- ***Pre- vs. post-development***

Model Checking is a verification method that is:

- model-based, automated, using a property-verification approach, mainly useful to verifying concurrent programs and reactive systems, typically in a post-development stage.

Program Verification (to be studied later), is:

- proof based, computer-assisted (partially-automated), mainly used for sequential, terminating programs

- Classical propositional and predicate calculi use a ***unique*** universe for interpreting formulas.
- In the 1950s Kripke introduced a type of semantic models where more (local) universes are possible
- There is a relation of ***accessibility*** between these universes and operators to express relationships between such universes, leading to various kinds of ***modalities***.
- When such operators are added, one gets ***modal logics***. When ***time*** is the parameter that causes the passing from one universe to another, one speaks about ***temporal logics***.

Programs (software) fit well in this framework:

- a universe corresponds to a state;
- the accessibility relation is given by the transition from one state to another;
- classic predicate logic may be used to specify relationships between variables in a state.

At this point we are lacking a mechanism to relate these universes (states). A variety of such mechanisms shall be introduced throughout this course.

Views of Time

We have the following characterizations of time:

- ***linear*** —a chain of time instances, or
- ***branching*** —several alternative future worlds may be possible at a given point in time ;

or

- ***discrete*** —the time is represented by the set of integers, or
- ***continuous*** —time is represented by the set of real numbers.

Next, we shall study ***Computation Tree Logic (CTL)*** which is a type of temporal logic using branching and discrete time.

For any model checking problem (based on CTL, or other logic), we are required to answer the question of whether

$$\mathcal{M}, s \models \Phi \,?$$

where

- $\mathcal{M}$ is an appropriate model for the given system, and s is a state of the model;
- Φ is a CTL formula intended to be satisfied by the system.

BNF definition of CTL:

$$\begin{aligned} \phi \;::=\; & \bot \mid \top \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \to \phi) \\ \mid\; & \mathrm{AX}\,\phi \mid \mathrm{EX}\,\phi \mid \mathrm{A}\,[\phi \mathrm{U} \phi] \mid \mathrm{E}\,[\phi\,\mathrm{U}\,\phi] \mid \mathrm{AG}\,\phi \mid \mathrm{EG}\,\phi \mid \mathrm{AF}\,\phi \mid \mathrm{EF}\,\phi \end{aligned}$$

The new connectives AX, EX, AU, EU, AG, EG, AF, and EF are called ***temporal connectives***.

Depth and Breadth Quantification

The temporal connectives use two letters:

- A and E to quantify over the breadth in a branching point:
  - A***ll alternatives*** in a branching point;
  - there E***xists at least one alternative*** in a branching point
- G and F to quantify along the paths:
  - all future states on a path, G***lobally***;
  - there exists at least one F***uture state*** along the path

Two more operators expressing properties along the paths are used:

- X to refer to the ***ne*X*t state*** in the path (this leads to the discrete feature of the time), and
- U —the U***ntil*** operator.

Priorities

Convention:

- the unary connectives (including AX, EX, AG, EG, AF, and EF) bind most tightly;
- next come $\wedge$ and $\vee$;
- lowest priority $\rightarrow$, AU and EU.

Examples

1 $\mathrm{EG}\, r$

2 $\mathrm{AG}\,(q \rightarrow \mathrm{EG}\, r)$

3 $\mathrm{A}\,[r \,\mathrm{U}\, q]$

4 $\mathrm{EF}\;\mathrm{E}[r \,\mathrm{U}\, q]$

5 $\mathrm{A}\,[p \,\mathrm{U}\, \mathrm{EF}\, r]$

6 $\mathrm{EF}\;\mathrm{EG}\, p \rightarrow \mathrm{AF}\, r$

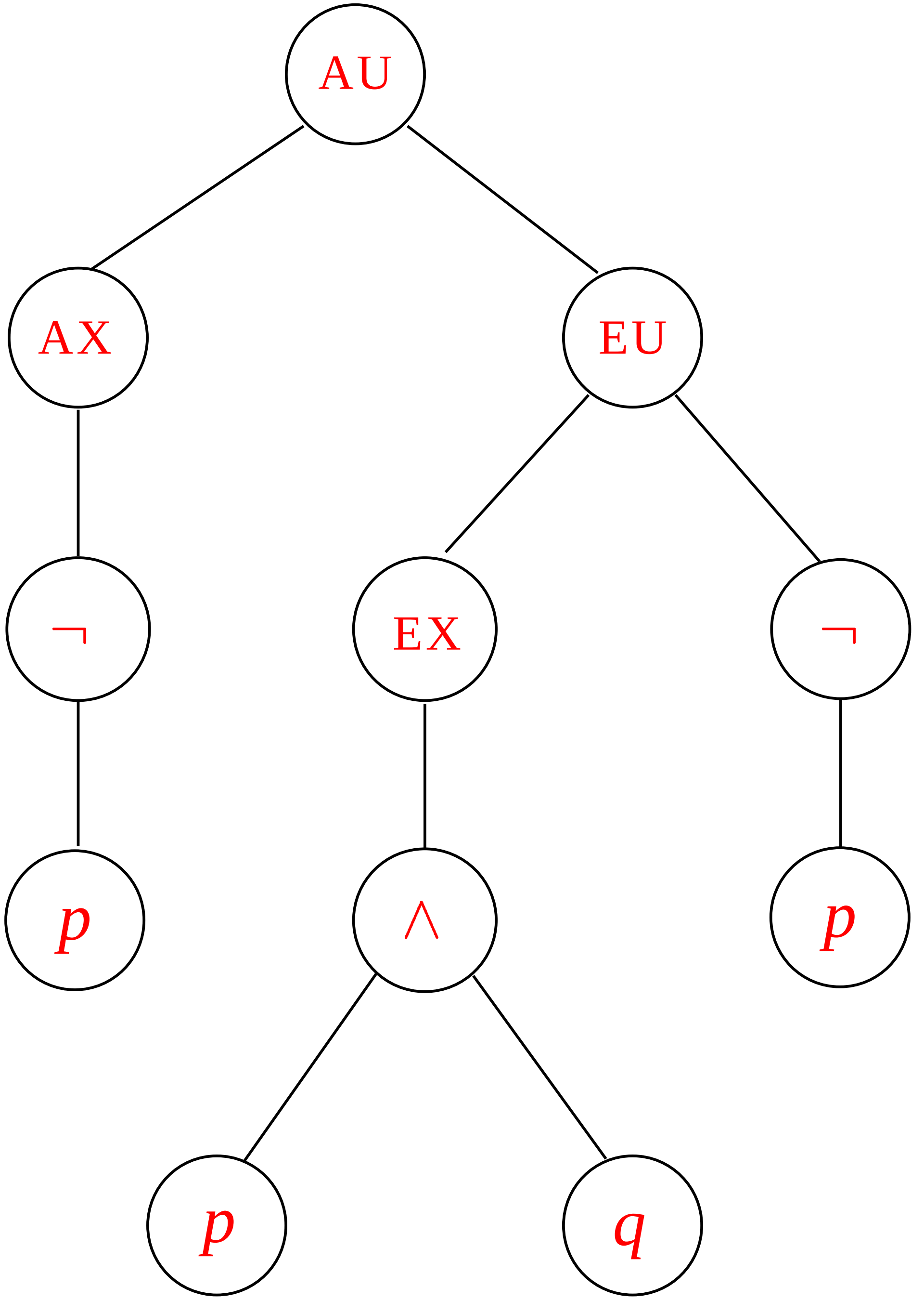
7 $\mathrm{AG}\;\mathrm{AF}\, r$

8 $\mathrm{A}\,[p_1 \,\mathrm{U}\, \mathrm{A}[p_2 \,\mathrm{U}\, p_3]]$

9 $\mathrm{E}\,[\mathrm{A}[p_1 \,\mathrm{U}\, p_2] \,\mathrm{U}\, p_3]$

10 $\mathrm{AG}(p \rightarrow \mathrm{A}\,[p \,\mathrm{U}\, (\neg p \wedge \mathrm{A}\,[\neg p \,\mathrm{U}\, q])])$

Examples (2)



The parse tree of the formula

$$\mathrm{A}[\mathrm{AX}\,\neg p\ \mathrm{U}\ \mathrm{E}[\mathrm{EX}(p \wedge q)\ \mathrm{U}\ \neg p]]$$

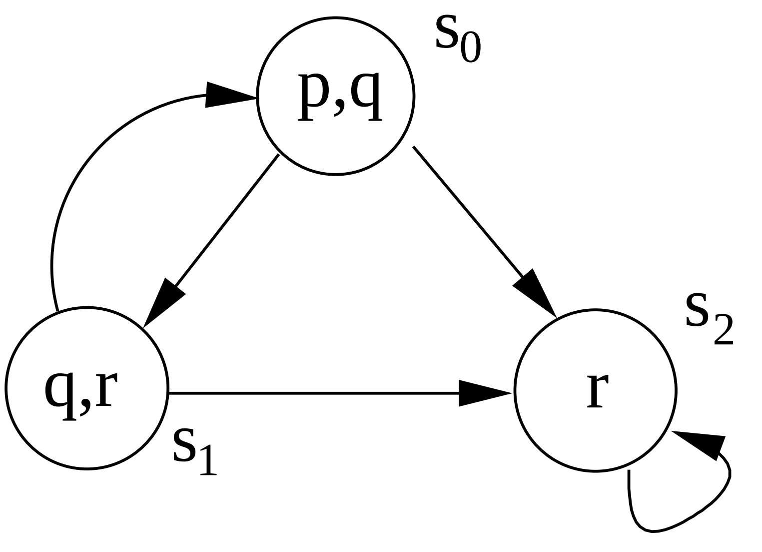
A ***model*** $\mathcal{M} = (S, \rightarrow, L)$ for CTL consists of

- a set of states S
- a binary relation $\rightarrow$ on S such that for every $s \in S$ there exists $s' \in S$ with $s \rightarrow s'$
- a labeling function $L : S \rightarrow \mathcal{P}(Atoms)$

The intuition is that L says which atoms are true in a state and $\rightarrow$ describes how the systems move from state to state.

Graphical description:

$S = \{s_0, s_1, s_2\}$
$\rightarrow = \{(s_0, s_1), (s_0, s_2), (s_1, s_0), (s_1, s_2), (s_2, s_2)\}$
$L(s_0) = \{p, q\}, L(s_1) = \{q, r\}, L(s_2) = \{r\}$



Let $\mathcal{M} = (S, \rightarrow, L)$ be a model for CTL, $s \in S$, and ϕ a CTL formula. The ***satisfaction relation***

$$\mathcal{M}, s \models \phi$$

is inductively defi ned by

1. $\mathcal{M}, s \models \top$ and $\mathcal{M}, s \not\models \bot$ for all $s \in S$;
2. $\mathcal{M}, s \models p$ iff $p \in L(s)$;
3. $\mathcal{M}, s \models \neg\phi$ iff $\mathcal{M}, s \not\models \phi$;
4. $\mathcal{M}, s \models \phi \wedge \psi$ iff $\mathcal{M}, s \models \phi$ and $\mathcal{M}, s \models \psi$;
5. $\mathcal{M}, s \models \phi \vee \psi$ iff $\mathcal{M}, s \models \phi$ or $\mathcal{M}, s \models \psi$;
6. $\mathcal{M}, s \models \phi \rightarrow \psi$ iff $\mathcal{M}, s \not\models \phi$ or $\mathcal{M}, s \models \psi$;
7. $\mathcal{M}, s \models \mathrm{AX}\,\phi$ iff for all s' such that $s \rightarrow s'$ we have $\mathcal{M}, s' \models \phi$;
8. $\mathcal{M}, s \models \mathrm{EX}\,\phi$ iff for some s' such that $s \rightarrow s'$ we have $\mathcal{M}, s' \models \phi$;

The Satisfaction Relation (2)

9. $\mathcal{M}, s \models \mathrm{AG}\ \phi$ iff for ***all paths*** $s = s_1 \rightarrow s_2 \rightarrow \ldots$ we have $\mathcal{M}, s_i \models \phi$, for ***all*** i;

10. $\mathcal{M}, s \models \mathrm{EG}\ \phi$ iff ***there exists a path*** $s = s_1 \rightarrow s_2 \rightarrow \ldots$ such that $\mathcal{M}, s_i \models \phi$, for ***all*** i;

11. $\mathcal{M}, s \models \mathrm{AF}\ \phi$ iff for ***all paths*** $s = s_1 \rightarrow s_2 \rightarrow \ldots$ we have $\mathcal{M}, s_i \models \phi$, for ***some*** i;

12. $\mathcal{M}, s \models \mathrm{EF}\ \phi$ iff ***there exists a path*** $s = s_1 \rightarrow s_2 \rightarrow \ldots$ such that $\mathcal{M}, s_i \models \phi$, for ***some*** i;

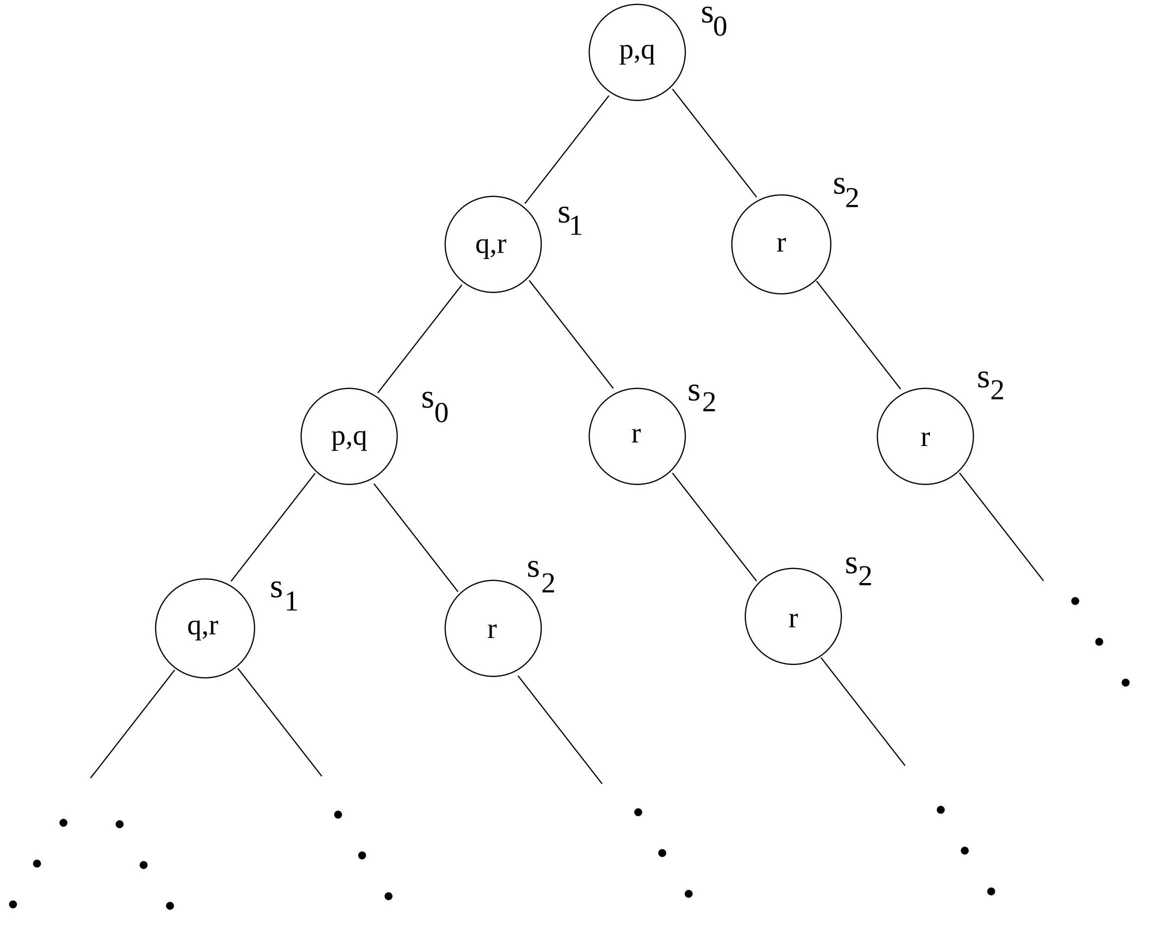
13. $\mathcal{M}, s \models \mathrm{A}[\phi\ \mathrm{U}\ \psi]$ iff for ***all paths*** $s = s_1 \rightarrow s_2 \rightarrow \ldots$ there exists an i such that $\mathcal{M}, s_i \models \psi$ and $\mathcal{M}, s_j \models \phi$ for all $j < i$;

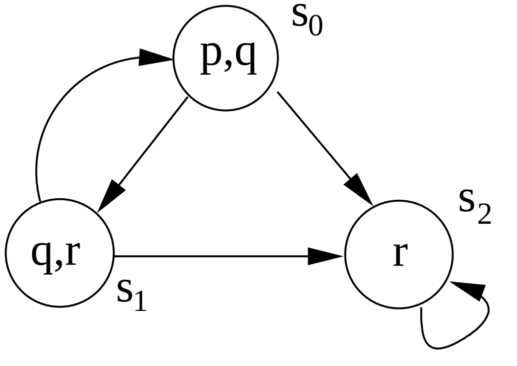
14. $\mathcal{M}, s \models \mathrm{E}[\phi\ \mathrm{U}\ \psi]$ iff ***there exists a path*** $s = s_1 \rightarrow s_2 \rightarrow \ldots$ and an i such that $\mathcal{M}, s_i \models \psi$ and $\mathcal{M}, s_j \models \phi$ for all $j < i$;

Comments

- Notice that 'the future' is the reflexive-transitive closure $\rightarrow^*$ of the (direct) accessibility relation $\rightarrow$.
- In common words:
  - the ***future contains the present*** and
  - ***a future of a future of*** t ***is a future of*** t.
- By unfolding [unwinding] the graph of a CTL model one gets and ***infinite tree***, whence ***'computation tree logic'***.

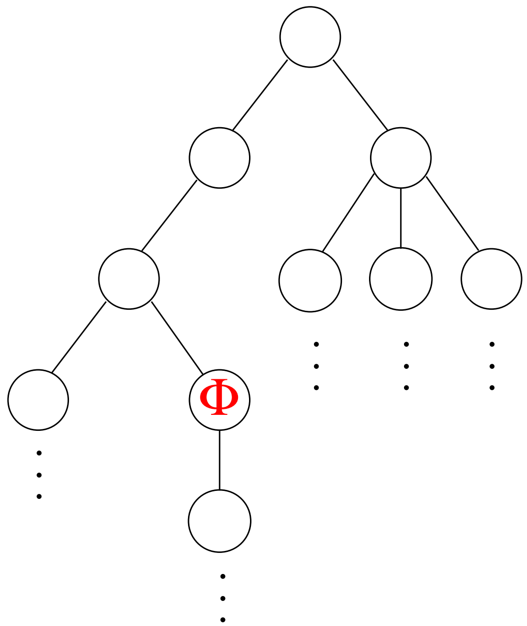
Unfolding



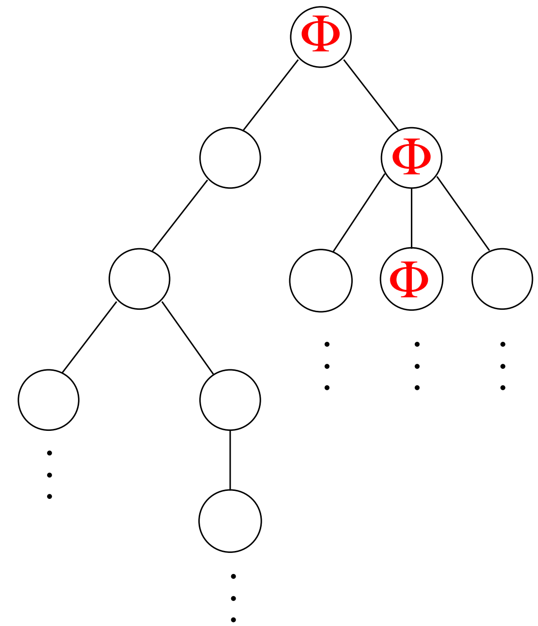


A CTL graph and its unfolding.

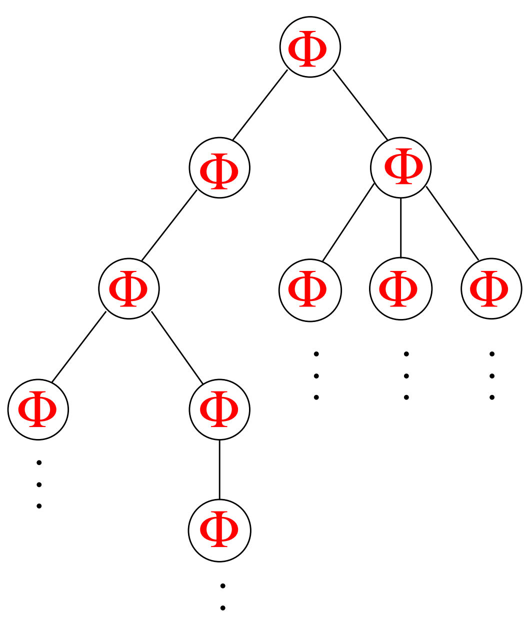
The Meaning of EF, EG, AG, and AF



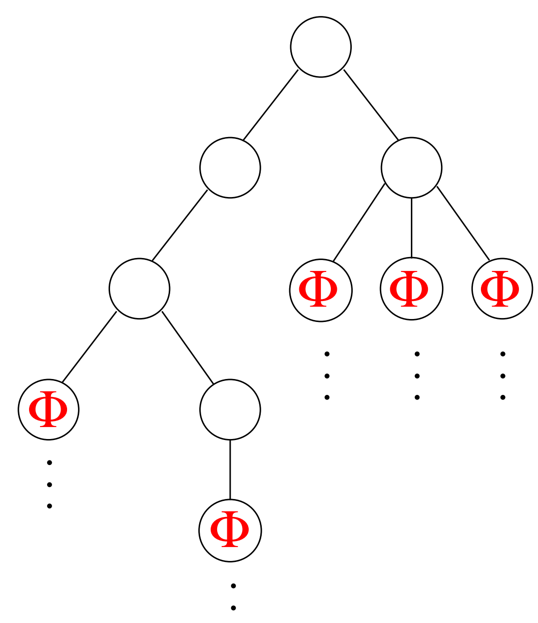
EF Φ



EG Φ

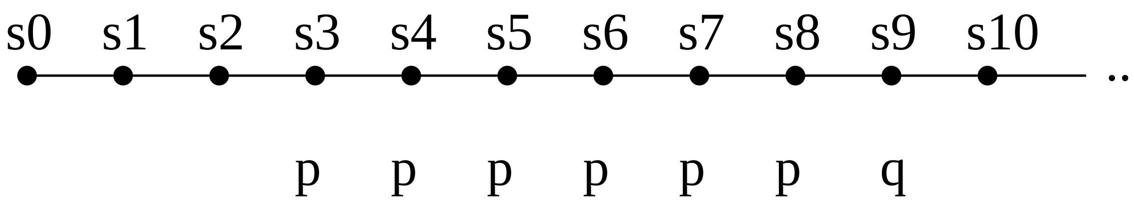


AG Φ



AF Φ

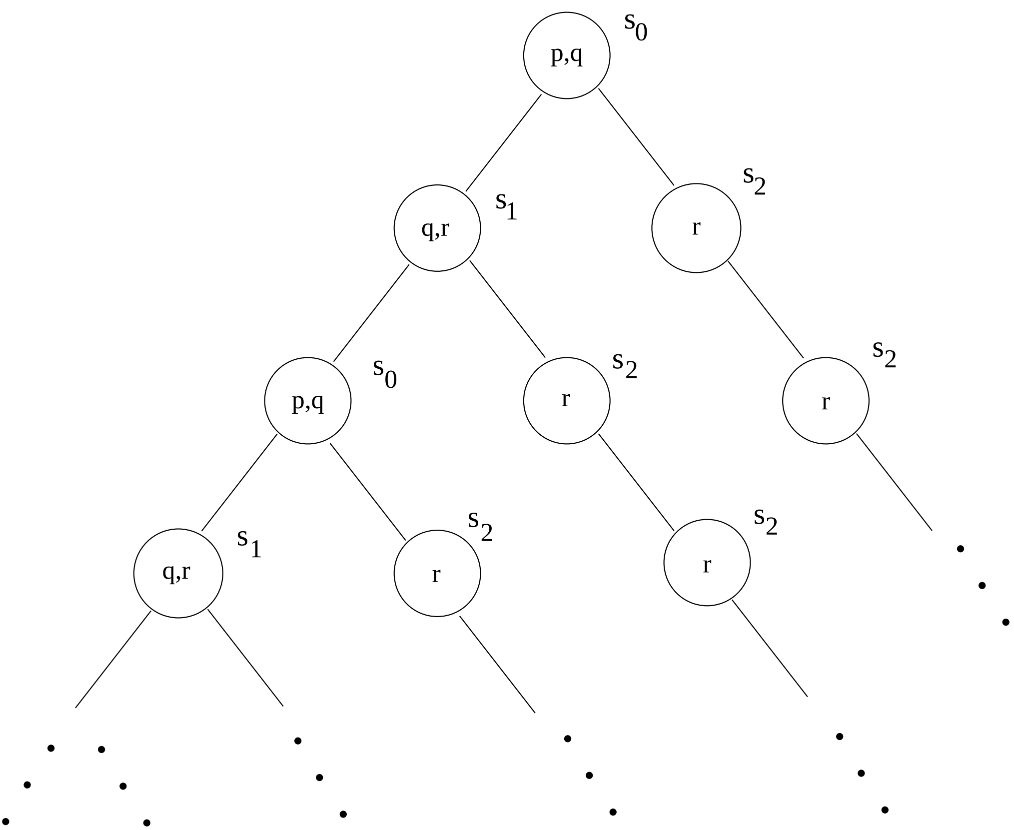
Until



Until $p \;\mathrm{U}\; q$ in a linear time model [or on a path in CTL].

The formula $p \;\mathrm{U}\; q$ holds in $s3$, but not in $s0$ (we suppose p holds only in the indicated states)

Examples



$\mathcal{M}, s0 \models p \wedge q$ - Yes

$\mathcal{M}, s0 \models \neg r$ - Yes

$\mathcal{M}, s0 \models \top$ - Yes

$\mathcal{M}, s0 \models \mathrm{EX}\,(q \wedge r)$ - Yes

$\mathcal{M}, s0 \models \neg\mathrm{AX}\,(q \wedge r)$ - Yes

$\mathcal{M}, s0 \models \neg\mathrm{EF}\,(p \wedge r)$ - Yes

$\mathcal{M}, s0 \models \mathrm{EG}\, r$ - No

$\mathcal{M}, s2 \models \mathrm{EG}\, r$ - Yes

$\mathcal{M}, s2 \models \mathrm{AG}\, r$ - Yes

$\mathcal{M}, s0 \models \mathrm{AF}\, r$ - Yes

$\mathcal{M}, s0 \models \mathrm{E}[(p \wedge q)\ \mathrm{U}\ r]$ - Yes

$\mathcal{M}, s0 \models \mathrm{A}[p\ \mathrm{U}\ r]$ - Yes

Practical Patterns of Specifications

Examples of practically relevant properties that may be checked:

- it is possible to reach a state where *started* holds, but *ready* not:

$$\text{EF}(started \wedge \neg ready)$$

- for any state, if a *request* occurs, then it will eventually be *acknowledged*:

$$\text{AG}(request \rightarrow \text{AF}\ acknowledged)$$

- a certain process is *enabled* infinitely often on every computation path:

$$\text{AG}(\text{AF}\ enabled)$$

- whatever happens, a certain process will eventually be permanently *deadlocked*:

$$\text{AF}(\text{AG}\ deadlocked)$$

Practical Patterns of Specifications (2)

- from any state it is possible to get to a *restart* state:

$$\mathrm{AG}(\mathrm{EF}\ restart)$$

- an upwards traveling elevator at the 2nd floor does not change its direction when has passengers going to the 5th floor:

$$\mathrm{AG}(floor = 2 \wedge direction = up \wedge ButtonPressed5$$
$$\rightarrow \mathrm{A}[direction = up\ \mathrm{U}\ floor = 5])$$

- the elevator can remain idle on the trird floor with its doors closed

$$\mathrm{AG}(floor = 3 \wedge idle \wedge door = closed$$
$$\rightarrow \mathrm{EG}(floor = 3 \wedge idle \wedge door = closed))$$

Definition: Two CTL formulas ϕ and ψ are ***semantically equivalent***, denoted $\phi \equiv \psi$ if any state in any model that satisfies one of them also satisfies the other.

Useful equivalences:

1 $\neg \mathrm{AF}\ \phi \equiv \mathrm{EG}\ \neg\phi$

2 $\neg \mathrm{EF}\ \phi \equiv \mathrm{AG}\ \neg\phi$

3 $\neg \mathrm{AX}\ \phi \equiv \mathrm{EX}\ \neg\phi$

4 $\mathrm{AF}\ \phi \equiv \mathrm{A}[\top\ \mathrm{U}\ \phi]$

5 $\mathrm{EF}\ \phi \equiv \mathrm{E}[\top\ \mathrm{U}\ \phi]$

Corollary (adequate sets of temporal connectives): The following sets of connectives are adequate for CTL (in the sense that each CTL formula may be transformed into an equivalent one using only those connectives):

- AU**,** EU ***and*** EX;
- EG**,** EU ***and*** EX; (hint for proof: $\mathrm{A}[\phi \; \mathrm{U} \; \psi] \equiv \neg(\mathrm{E}[\neg\psi \; \mathrm{U} \; (\neg\phi \wedge \neg\psi)] \vee \mathrm{EG} \, \neg\psi))$
- AG**,** AU ***and*** AX;
- AF**,** EU ***and*** AX

More useful equivalences (fixed-point definitions)

6 $\mathrm{AG}\ \phi \equiv \phi \wedge \mathrm{AX}\ \mathrm{AG}\ \phi$

7 $\mathrm{EG}\ \phi \equiv \phi \wedge \mathrm{EX}\ \mathrm{EG}\ \phi$

8 $\mathrm{AF}\ \phi \equiv \phi \vee \mathrm{AX}\ \mathrm{AF}\ \phi$

9 $\mathrm{EF}\ \phi \equiv \phi \vee \mathrm{EX}\ \mathrm{EF}\ \phi$

10 $\mathrm{A}[\phi\ \mathrm{U}\ \psi] \equiv \psi \vee (\phi \wedge \mathrm{AX}\ \mathrm{A}[\phi\ \mathrm{U}\ \psi])$

11 $\mathrm{E}[\phi\ \mathrm{U}\ \psi] \equiv \psi \vee (\phi \wedge \mathrm{EX}\ \mathrm{E}[\phi\ \mathrm{U}\ \psi])$

A mechnism for solving such ***fixed-point*** equations $Y = \phi \wedge \mathrm{AX}\ Y$ and the next operators AX and EX are sufficient to represent all temporal logic operators.

Goal: to develop protocols for accessing some critical sections such that only one process can be in its critical section at a time. A collection of desirable properties is:

Safety: the protocol allows only one process to be in its critical section at any time

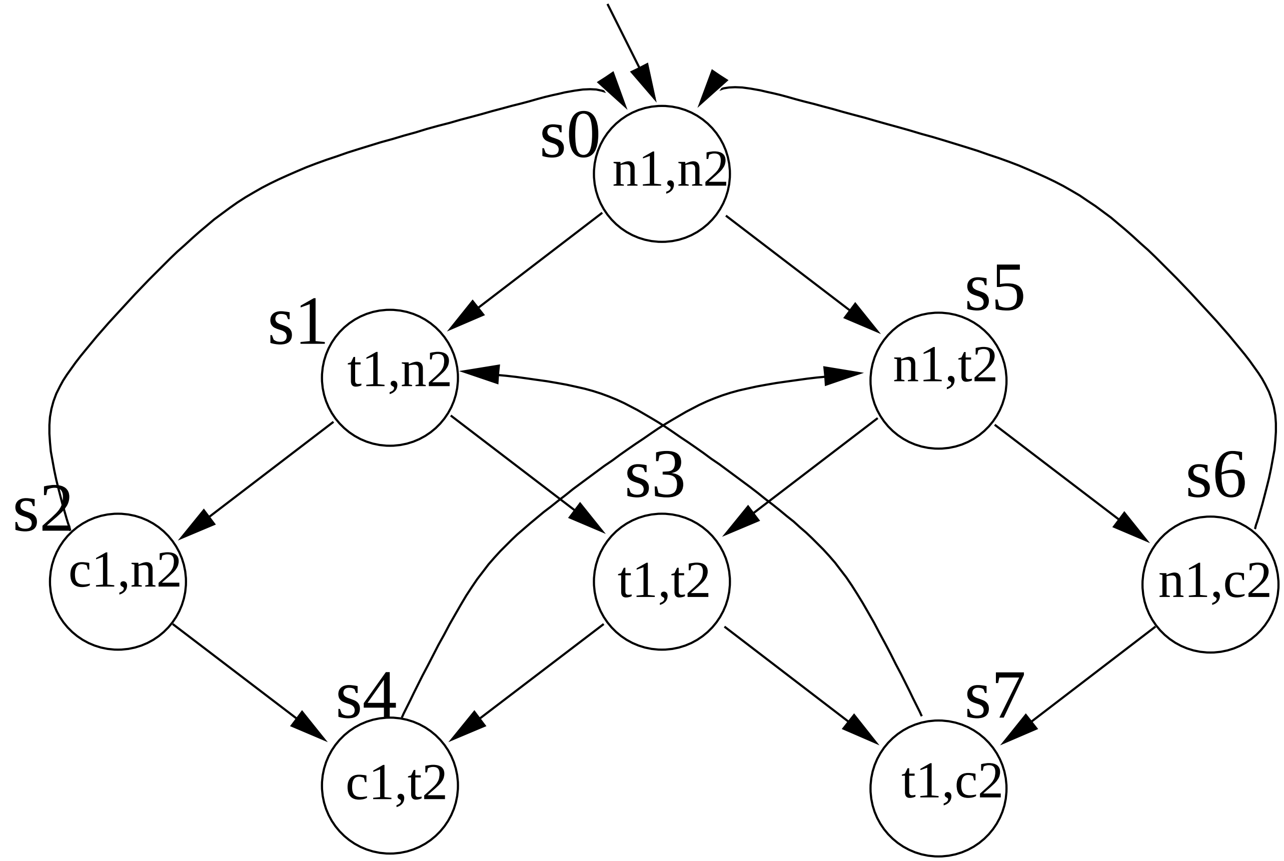
Liveness: whenever any process wants to enter its critical section, it will eventually be permitted to do so

Non-blocking: a process can always request to enter its critical section

No strict sequencing: Processes need not enter their critical section in strict sequence

Mutual Exclusion (2)

A simple model MUT1:



The system consists of two processes $P1$ and $P2$, each making a loop $n \to t \to c \to \ldots$ (noncritical $\to$ trying $\to$ critical $\to \ldots$).

The system's behaviour is the product (interleaving) of the behaviours of $P1$ and $P2$, but the state $(c1, c2)$ is excluded.

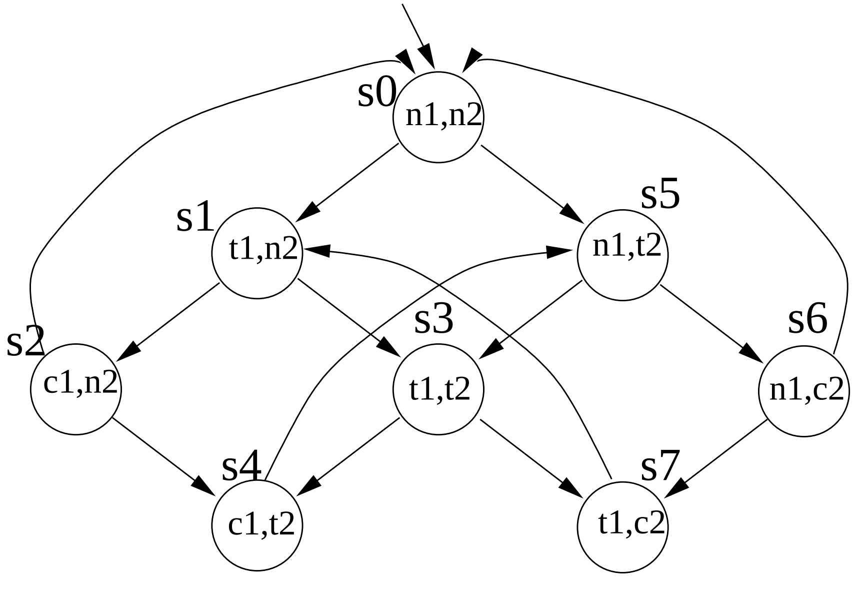
Safety: $\phi_1 =_{def} \mathrm{AG}\ \neg(c1 \wedge c2)$ —satisfied in each state;

Liveness: $\phi_2 =_{def} \mathrm{AG}(t1 \rightarrow \mathrm{AF}\ c1)$ —not satisfied in the initial state $s0$; e.g., $s1$ is accessible, $t1$ is true, but there is a path $s1 \rightarrow s3 \rightarrow s7 \rightarrow s1 \rightarrow \ldots$ where $c1$ is always false;

Non-blocking: $\phi_3 =_{def} \mathrm{AG}(n1 \rightarrow \mathrm{EX}\ t1)$ —true

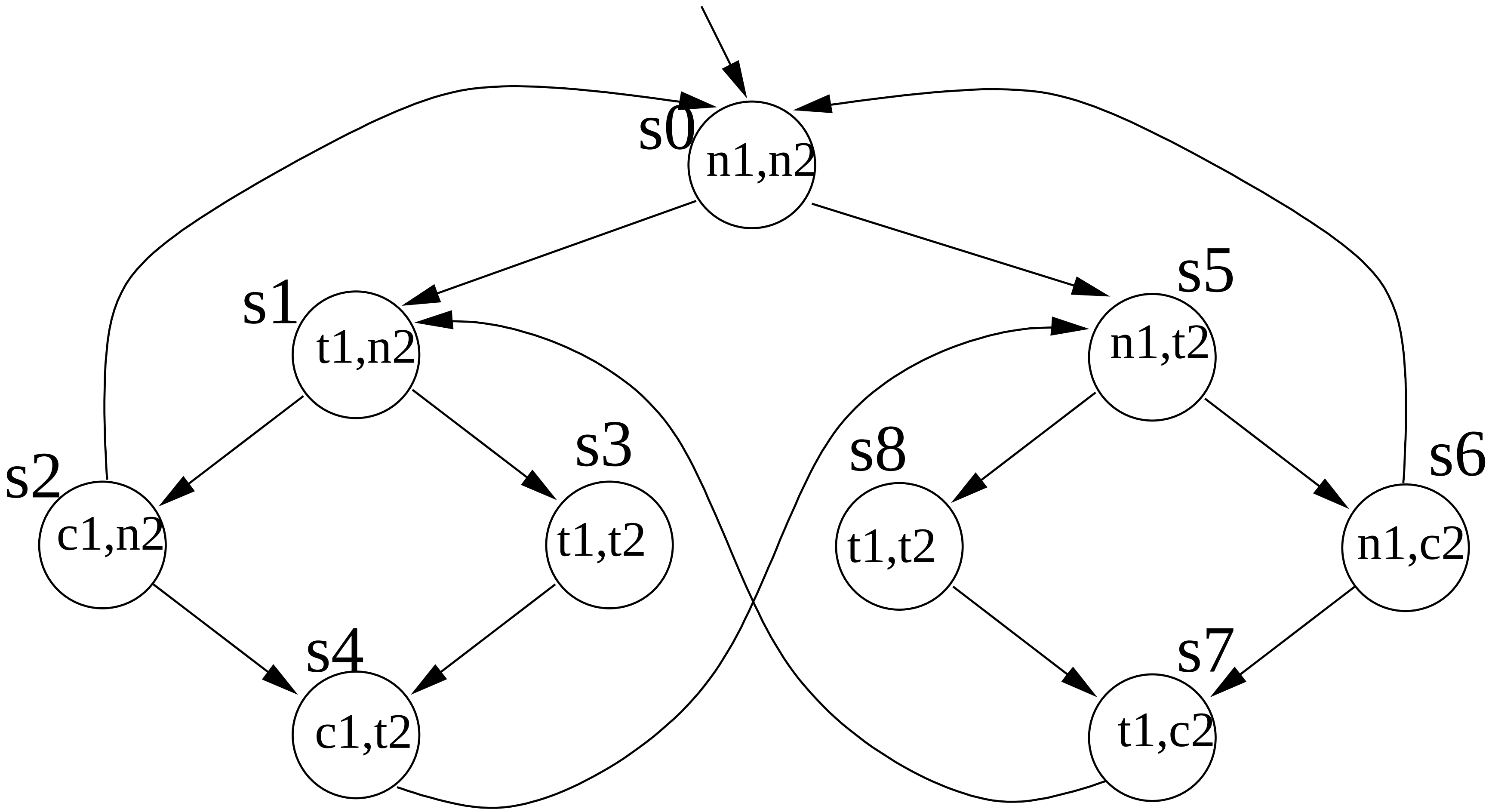
No strict sequencing:

$\phi_4 =_{def} \mathrm{EF}(c1 \wedge \mathrm{E}[c1\ \mathrm{U}\ (\neg c1 \wedge \mathrm{E}[\neg c2\ \mathrm{U}\ c1])])$ —true



Mutual Exclusion (4)

A second model MUT2:



- This model is obtained by splitting state $s3$ of MUT1 in two different states $s3$ and $s8$.
- By spliting $s3$ into two states we are able to identify which process was the first asking to access its critical section: if $P1$ was the first, the the resulting state is $s3$, otherwise $s8$.

Fact: All four properties (i.e., formuls ϕ_1 to ϕ_4) are valid in MUT2.

Model Checking Algorithms

Recall that the problem to be solved by model checking is

(A) "Given a model $\mathcal{M}$, a CTL formula ϕ, and ***a state*** s, does $\mathcal{M}, s \models \phi$ hold?", where

- $\mathcal{M}$ is a model of the system and s is a state of the model;
- ϕ is a CTL formula intended to be satisfied by the system

What is the model used for the system? Typically:

> ***"A system is represented by a finite transition system (usually, a huge labeled directed graph, often with milions of states)."***

The infinite trees obtained by unfolding such graphs are useful to develop an intuition of the reasoning process, but not to be used on our finite computers.

A Model Checking Result

Given $\mathcal{M}, s$, and ϕ, the result returned by a model checker is either

(1) yes: $\mathcal{M}, s \models \phi$ or

(2) no: $\mathcal{M}, s \not\models \phi$

but, quite useful, in the latter case most of model checkers return a trace/path which invalidates ϕ, as well (a counterexample).

Alternative problem:

(B) Given a model $\mathcal{M}$ and a CTL formula ϕ find ***all states*** s of the model which satisfy ϕ

These two problems are obviously equivalent: once one is able to develop algorithms to solve one of them, the other is solved, as well. We will be mainly concerned with the ***latter*** problem B.

Reduced Set of CTL Connectives

We are starting with a version using the following reduced set of CTL connectives

$$\Gamma = \{\bot,\ \neg,\ \wedge,\ \mathrm{AF},\ \mathrm{EU},\ \mathrm{EX}\}$$

where:

- $\bot, \neg$, and $\wedge$ are used for the propositional part
- $\mathrm{AF},\ \mathrm{EU}$, and EX are used for the temporal part

Hence, there is a ***preprocessing*** procedure to:

1. check the CTL syntax correctness of the given formula ϕ and
2. translate it in a formula $\mathrm{TRANSLATE}(\phi)$ written with connectives in Γ, only.

In the sequel, we suppose ϕ to be in CTL Γ-format.

The idea of this algorithm is to:

- decompose formula ϕ in pieces (sub-formulas) and apply a structural induction to label the graph with sub-formulas of ϕ (the intuition is that *a formula that labels a state is true in that state*)
- for each such sub-formula, parse the graph to infer the truth in a state according to the meaning of the connectives and the truth values of its sub-formulas

In 2, one may need to know the values of sub-formulas in possibly many different states; this is the case for temporal operators, but not for the propositional ones.

Input: a CTL model $\mathcal{M} = (S, \rightarrow, L)$ and a CTL formula ϕ
(in Γ-format)

Output: the set of states of $\mathcal{M}$ which satisfy ϕ

1. $\perp$: no states are labeled with $\perp$
2. p: label with p all states s such that $p \in L(s)$
3. $\neg\phi_1$: label s with $\neg\phi_1$ if s is not already labeled with ϕ_1
4. $\phi_1 \wedge \phi_2$: label s with $\phi_1 \wedge \phi_2$ if s is already labeled both with ϕ_1 and ϕ_2
5. $\mathrm{EX}\ \phi_1$: label s with $\mathrm{EX}\ \phi_1$ if one of its successors is already labeled with ϕ_1

The Labeling Algorithm (3)

6. $\mathrm{AF}\ \phi_1$:

   a. (initial marking) label any s with $\mathrm{AF}\ \phi_1$ if s is already labeled with ϕ_1
   b. (repeated marking) label any s with $\mathrm{AF}\ \phi_1$ if all successor states of s are already labeled with $\mathrm{AF}\ \phi_1$
   c. repeat (2) until there are no change

7. $\mathrm{E}[\phi_1\ \mathrm{U}\ \phi_2]$:

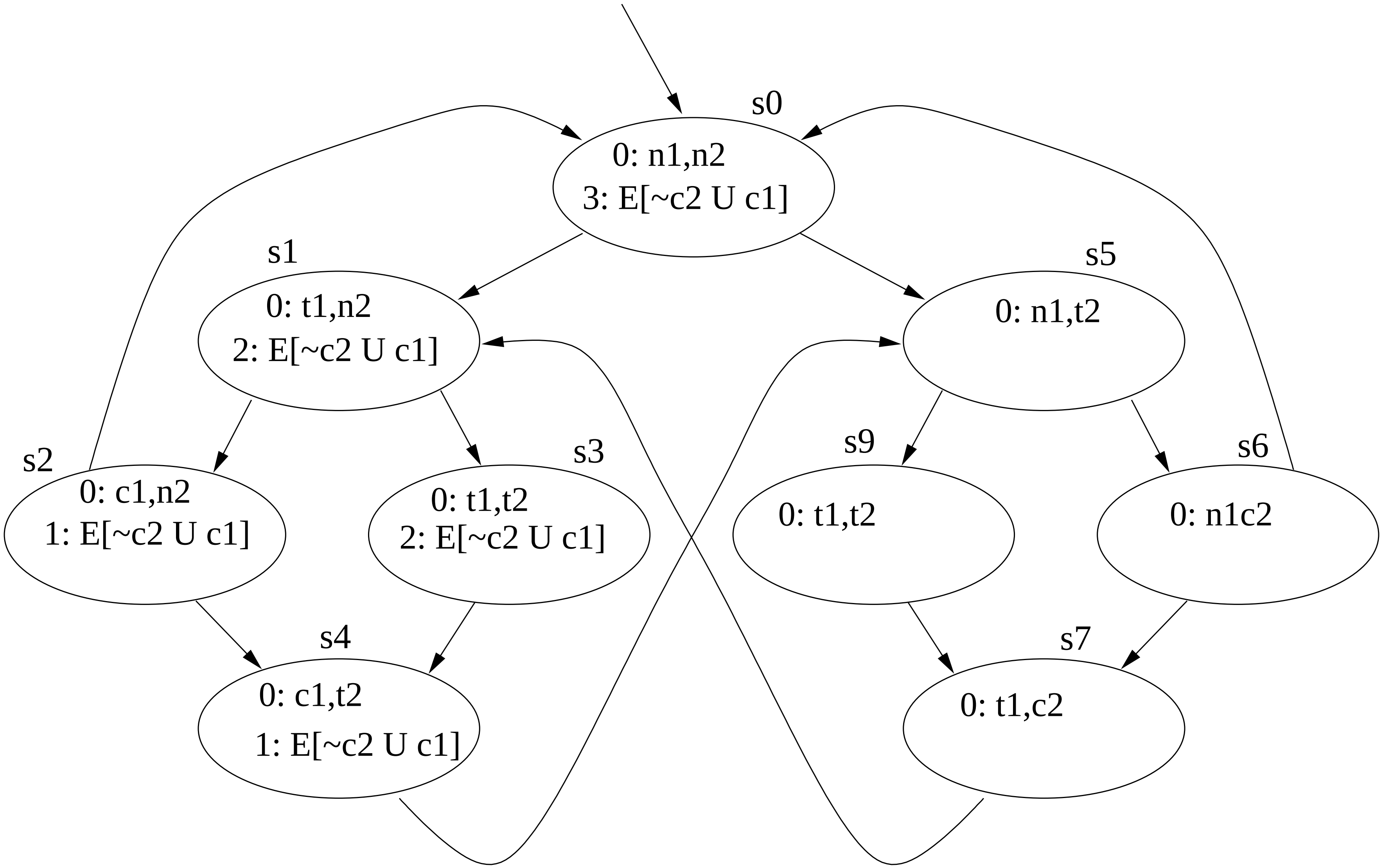
   a. (initial marking) label any s with $\mathrm{E}[\phi_1\ \mathrm{U}\ \phi_2]$ if s is already labeled with ϕ_2
   b. (repeated marking) label any s with $\mathrm{E}[\phi_1\ \mathrm{U}\ \phi_2]$ if s is already labeled with ϕ_1 and at least one of its successor states is already labeled with $\mathrm{E}[\phi_1\ \mathrm{U}\ \phi_2]$
   c. repeat (2) until there is no change

A rough analysis of the algorithm shows that it has the worse time complexity

$$O(k \cdot m \cdot (m+n))$$

where k is the number of connectives of the formula, m is the number of the states of the model, and n is the number of the transitions of the model.

Checking $\mathrm{E}[\neg c_2 \ \mathrm{U} \ c_1]$ in the second mutual exclusion model MUT2:



EG **Directly**

EG may be handled directly as follows:

6' EG ϕ_1:

0. label *all* states *s* with EG ϕ_1
1. (initial de-marking) if ϕ_1 does not hold in *s* then *delete* the label EG ϕ_1
2. (repeated de-marking) *delete* the label EG ϕ_1 from any state *s* if none of its successor states is labeled with EG ϕ_1
3. repeat (2) until there are no change

This different approach is based on the following greatest fixed-point characterization of EG

$$\mathrm{EG}\ \phi \equiv \phi \wedge \mathrm{EX}\ \mathrm{EG}\ \phi$$

An Improved Variant

- Use EX, EU, and EG instead of EX, EU, and AF
- handle EX and EU as before (using backwards breadth-first search)
- for $\mathrm{EG}\ \phi$
  - restrict to states satisfying ϕ
  - find SCCs (maximal strongly connected components; these are maximal regions such that any vertex is connected to any other vertex in the region)
  - use backwards breadth-first searching on the restricted graph to find any state that can reach an SCC

Complexity is reduced to $O(k \cdot (m+n))$ (k, m, n as before).

Pseudo-Code

function $\text{SAT}(\phi)$:

/* precondition: ϕ is an arbitrary CTL formula */

/* postcondition: $\text{SAT}(\phi)$ returns the set of states satisfying ϕ */

begin function

case

ϕ is $\top$: **return** S

ϕ is $\bot$: **return** $\emptyset$

ϕ is atomic formula: **return** $\{s \in S \mid \phi \in L(s)\}$

ϕ is $\neg\phi_1$: **return** $S \setminus \text{SAT}(\phi_1)$

ϕ is $\phi_1 \wedge \phi_2$: **return** $\text{SAT}(\phi_1) \cap \text{SAT}(\phi_2)$

ϕ is $\phi_1 \vee \phi_2$: **return** $\text{SAT}(\phi_1) \cup \text{SAT}(\phi_2)$

ϕ is $\phi_1 \rightarrow \phi_2$: **return** $\text{SAT}(\neg\phi_1 \vee \phi_2)$

Pseudo-Code (2)

(...cont.)

ϕ is $\mathrm{AX}\phi_1$: **return** SAT$(\neg\mathrm{EX}\neg\phi_1)$

ϕ is $\mathrm{EX}\phi_1$: **return** SAT$_{EX}(\phi_1)$

ϕ is $\mathrm{A}[\phi_1 \mathrm{U} \phi_2]$:

return SAT$(\neg(\mathrm{E}[\neg\phi_1 \mathrm{U}(\neg\phi_1 \wedge \neg\phi_2)] \vee \mathrm{EG}\neg\phi_2))$

ϕ is $\mathrm{E}[\phi_1 \mathrm{U} \phi_2]$: **return** SAT$_{EU}(\phi_1, \phi_2)$

ϕ is $\mathrm{EF}\phi_1$: **return** SAT$(\mathrm{E}[\top\ \mathrm{U}\ \phi_1])$

ϕ is $\mathrm{EG}\phi_1$: **return** SAT$(\neg\mathrm{AF}\neg\phi_1])$

ϕ is $\mathrm{AF}\phi_1$: **return** SAT$_{AF}(\phi_1)$

ϕ is $\mathrm{AG}\phi_1$: **return** SAT$(\neg\mathrm{EF}\neg\phi_1)$

end case

end function

Pseudo-Code (3)

function $\mathrm{SAT}_{EX}(\phi)$:

/* pre: ϕ is an arbitrary CTL formula */

/* post: $\mathrm{SAT}_{EX}(\phi)$ returns the set of states satisfying $\mathrm{EX}\ \phi$ */

local var X, Y

begin

$X := \mathrm{SAT}(\phi)$;

$Y := \{s_0 \in S \mid s_0 \to s_1$ for some $s_1 \in X\}$;

return Y

end

Pseudo-Code (4)

function $\text{SAT}_{AF}(\phi)$:
/* pre: ϕ is an arbitrary CTL formula */
/* post: $\text{SAT}_{AF}(\phi)$ returns the set of states satisfying $\text{AF}\ \phi$ */
local var X, Y
begin
    $X := S$;
    $Y := \text{SAT}(\phi)$;
    repeat until $X = Y$
        begin
            $X := Y$;
            $Y := Y \cup \{s \in S \mid$ for all s' with $s \to s'$ we have $s' \in Y\}$;
        end
    return Y
end

function $\mathrm{SAT}_{EU}(\phi,\psi)$:
/* pre: ϕ is an arbitrary CTL formula */
/* post: $\mathrm{SAT}_{EU}(\phi,\psi)$ returns the set of states satisfying $\mathrm{E}[\phi \mathrm{U} \psi]$ */
local var W, X, Y
begin
$W := \mathrm{SAT}(\phi)$;
$X := S$;
$Y := \mathrm{SAT}(\psi)$;
repeat until $X = Y$
begin
$X := Y$;
$Y := Y \cup (W \cap \{s \in S \mid \text{exists } s' \text{ such that } s \rightarrow s' \text{ and } s' \in Y\}$;
end
return Y
end

- the labeling algorithm is quite efficient [linear in the size of the model]
- ... but the model itself may be large, exponential in the number of the components (running in parallel 10 threads each of them having 10 states results in a systems with $10^{10} = 10,000,000,000$ states!)
- the tendency of the state space to become very large is commonly referred to as the ***state explosion*** problem
- the state explosion problem is mainly ***unsolved*** - no general solution is known at the moment

Dealing With the State Explosion Code

The problem is general unsolved. The following techniques were developed to overcome it in certain particular cases:

1. ***efficient data structures*** - e.g., *ordered binary decision diagrams OBDDs* (OBDDs are used to represents sets of states, not individual states)
2. ***abstraction*** - one may abstract away variables in the model that are not relevant for the formula being checked
3. ***partial order reduction*** - different runnings may be equivalent as far as the formula to be checked is concerned; partial order reduction check one trace from such a class only
4. ***induction*** - this technique is used when a large number of processes is considered
5. ***composition*** - try to split the problem in small parts to be separately checked