

CS3234 - Tutorial 1, Solutions

1.

1. p = barometer falls; q = it will rain; r = it will snow

$$p \rightarrow (q \wedge \neg r) \vee (r \wedge \neg q)$$

2. p = Dick met Jane yesterday; q = they had a cup of coffee together; r = they took a walk in the park

$$p \rightarrow q \vee r$$

3. p = today it will rain; q = today it will shine

$$(p \wedge \neg q) \vee (\neg p \wedge q)$$

2.

1. $((\neg p) \wedge q) \rightarrow r$
2. $(p \rightarrow q) \wedge (\neg((r \vee p) \rightarrow q))$
3. $(p \vee q) \rightarrow ((\neg p) \wedge r)$

3.

1. $p \rightarrow q, r \rightarrow s \vdash p \vee r \rightarrow q \vee s$

1	$p \rightarrow q$	premise
2	$r \rightarrow s$	premise
3	$p \vee r$	assumption
4	p	assumption
5	q	\rightarrow e 4,1
6	$q \vee s$	\vee i 5
7	r	assumption
8	s	\rightarrow e 7,2
9	$q \vee s$	\vee i 8
10	$q \vee s$	\vee e 3,4-6,7-9
11	$p \vee r \rightarrow q \vee s$	\rightarrow i 3-10

2. $\vdash (p \wedge q) \rightarrow p$

1	$p \wedge q$	assumption
2	p	\wedge e 1
3	$(p \wedge q) \rightarrow p$	\rightarrow i 1-2

3. $p \vdash (p \rightarrow q) \rightarrow q$

1	p	premise
2	$p \rightarrow q$	assumption
3	q	\rightarrow e 1,2
4	$(p \rightarrow q) \rightarrow q$	\rightarrow i 2-3

4. $\vdash (p \rightarrow q) \vee (q \rightarrow p)$

1	$p \vee \neg p$	LEM
2	p	assumption
3	q	assumption
4	p	copy 2
5	$q \rightarrow p$	\rightarrow i 3-4
6	$(p \rightarrow q) \vee (q \rightarrow p)$	\vee i2 5
7	$\neg p$	assumption
8	p	assumption
9	\perp	\neg e 7,8
10	q	\perp e 9
11	$p \rightarrow q$	\rightarrow i 8-10
12	$(p \rightarrow q) \vee (q \rightarrow p)$	\vee i1 11
13	$(p \rightarrow q) \vee (q \rightarrow p)$	\vee e 1,2-6,7-12

- o 2.-5. represents proof for $p \vdash (q \rightarrow p)$;
- 7.-11. represents proof for $\neg p \vdash (p \rightarrow q)$

5. $\neg(\neg p \vee q) \vdash p$

1	$\neg(\neg p \vee q)$	premise
2	$p \vee \neg p$	LEM
3	p	assumption
4	$\neg p$	assumption
5	$\neg p \vee q$	\vee i1 4
6	\perp	\neg e 1,5
7	$\neg\neg p$	\neg i 4-6
8	p	$\neg\neg$ e 7
9	p	\vee e 2,3,4-8

6. $\vdash (p \rightarrow q) \vee (q \rightarrow r)$

1	$q \vee \neg q$	LEM
2	q	assumption
3	p	assumption
4	q	copy 2
5	$p \rightarrow q$	\rightarrow i 3-4
6	$(p \rightarrow q) \vee (q \rightarrow r)$	\vee i1 5
7	$\neg q$	assumption
8	q	assumption
9	\perp	\neg e 7,8
10	r	\perp e 9
11	$q \rightarrow r$	\rightarrow i 8-10
12	$(p \rightarrow q) \vee (q \rightarrow r)$	\vee i2 11
13	$(p \rightarrow q) \vee (q \rightarrow r)$	\vee e 1,2-6,7-12

- o 2.-5. represents proof for $q \vdash (p \rightarrow q)$;
- 7.-11. represents proof for $\neg q \vdash (q \rightarrow r)$

7. $\vdash (p \rightarrow q) \rightarrow ((\neg p \rightarrow q) \rightarrow q)$

1	$p \rightarrow q$	assumption
2	$\neg p \rightarrow q$	assumption
3	$p \vee \neg p$	LEM
4	p	assumption
5	q	\rightarrow e 4,1
6	$\neg p$	assumption
7	q	\rightarrow e 6,2
8	q	\vee e 3,4-5,6-7
9	$(\neg p \rightarrow q) \rightarrow q$	\rightarrow i 2-8
10	$(p \rightarrow q) \rightarrow ((\neg p \rightarrow q) \rightarrow q)$	\rightarrow i 1-9

8. $p \rightarrow q, r \rightarrow s \vdash p \wedge r \rightarrow q \wedge s$

1	$p \rightarrow q$	premise
2	$r \rightarrow s$	premise
3	$p \wedge r$	assumption
4	p	\wedge e1 3
5	q	\rightarrow e 4,1
6	r	\wedge e2 3
7	s	\rightarrow e 6,2
8	$q \wedge s$	\wedge i 5,7
9	$p \wedge r \rightarrow q \wedge s$	\rightarrow i 3-8

9. $p \rightarrow (q \vee r), q \rightarrow s, r \rightarrow s \vdash p \rightarrow s$

1	$p \rightarrow (q \vee r)$	premise
2	$q \rightarrow s$	premise
3	$r \rightarrow s$	premise
4	p	assumption
5	$q \vee r$	$\rightarrow e$ 4,1
6	q	assumption
7	s	$\rightarrow e$ 6,2
8	r	assumption
9	s	$\rightarrow e$ 8,3
10	s	$\vee e$ 5,6-7,8-9
11	$p \rightarrow s$	$\rightarrow i$ 4-10

10. $(p \wedge q) \vee (p \wedge r) \vdash p \wedge (q \vee r)$

1	$(p \wedge q) \vee (p \wedge r)$	premise
2	$p \wedge q$	assumption
3	p	$\wedge e1$ 2
4	q	$\wedge e2$ 2
5	$q \vee r$	$\vee i1$ 4
6	$p \wedge (q \vee r)$	$\wedge i$ 3,5
7	$p \wedge r$	assumption
8	p	$\wedge e1$ 7
9	r	$\wedge e2$ 7
10	$q \vee r$	$\vee i2$ 9
11	$p \wedge (q \vee r)$	$\wedge i$ 8,10
12	$p \wedge (q \vee r)$	$\vee e$ 1,2-6,7-11

11. $q \rightarrow (p \rightarrow r), \neg r, q \vdash \neg p$

1	$q \rightarrow (p \rightarrow r)$	premise
2	$\neg r$	premise
3	q	premise
4	$p \rightarrow r$	$\rightarrow e$ 3,1
5	$\neg p$	MT 4,2

12. $p \rightarrow q \wedge r \vdash (p \rightarrow q) \wedge (p \rightarrow r)$

1	$p \rightarrow q \wedge r$	premise
2	p	assumption
3	$q \wedge r$	$\rightarrow e$ 2,1
4	q	$\wedge e$ 3
5	$p \rightarrow q$	$\rightarrow i$ 2-4
6	p	assumption
7	$q \wedge r$	$\rightarrow e$ 6,1
8	r	$\wedge e$ 7
9	$p \rightarrow r$	$\rightarrow i$ 6-8
10	$(p \rightarrow q) \wedge (p \rightarrow r)$	$\wedge i$ 5,9

13. $\neg(p \wedge q) \vdash \neg p \vee \neg q$

1	$\neg(p \wedge q)$	premise
2	$p \vee \neg p$	LEM
3	p	assumption
4	$q \vee \neg q$	LEM
5	q	assumption
6	$p \wedge q$	\wedge i 3,5
7	\perp	\neg e 6,1
8	$\neg p \vee \neg q$	\perp e 7
9	$\neg q$	assumption
10	$\neg p \vee \neg q$	\vee i2 9
11	$\neg p \vee \neg q$	\vee e 4,5-8,9-10
12	$\neg p$	assumption
13	$\neg p \vee \neg q$	\vee i1 12
14	$\neg p \vee \neg q$	\vee e 2,3-11,12-13

14. $p \rightarrow (q \vee r), \neg q, \neg r \vdash \neg p$

1	$p \rightarrow (q \vee r)$	premise
2	$\neg q$	premise
3	$\neg r$	premise
4	$p \vee \neg p$	LEM
5	p	assumption
6	$q \vee r$	\rightarrow e 5,1
7	q	assumption
8	\perp	\neg e 7,2
9	r	assumption
10	\perp	\neg e 9,3
11	\perp	\vee e 6,7-8,9-10
12	$\neg p$	\neg i 5-11

4.

$$\leftrightarrow i \quad \frac{\begin{array}{|c|} \hline \phi \\ \vdots \\ \psi \\ \hline \end{array} \quad \begin{array}{|c|} \hline \psi \\ \vdots \\ \phi \\ \hline \end{array}}{\phi \leftrightarrow \psi}$$

1	ϕ	premise
2	\vdots	
3	ψ	
4	ψ	premise
5	\vdots	
6	ϕ	
7	$\phi \rightarrow \psi$	$\rightarrow i$ 1-3
8	$\psi \rightarrow \phi$	$\rightarrow i$ 4-6
9	$(\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$	$\wedge i$ 7,8
10	$\phi \leftrightarrow \psi$	copy 9

$$\leftrightarrow e1 \quad \frac{\phi \quad \phi \leftrightarrow \psi}{\psi}$$

1	ϕ	premise
2	$\phi \leftrightarrow \psi$	premise
3	$(\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$	copy 2
4	$\phi \rightarrow \psi$	$\wedge e1$ 3
5	ψ	$\rightarrow e$ 1,4

$\leftrightarrow e2$ $\frac{\psi \quad \phi \leftrightarrow \psi}{\phi}$

1	ψ	premise
2	$\phi \leftrightarrow \psi$	premise
3	$(\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$	copy 2
4	$\psi \rightarrow \phi$	$\wedge e2$ 3
5	ϕ	$\rightarrow e$ 1,4