

CS3234 — Tutorial 2 questions

1. **Parse trees:** For each of the following formulas

1. $\neg((\neg q \wedge (p \rightarrow r)) \wedge (r \rightarrow q))$
2. $((s \rightarrow (r \vee l)) \vee ((\neg q) \wedge r)) \rightarrow ((\neg(p \rightarrow s)) \rightarrow r)$
3. $((p \rightarrow \neg q) \vee (p \wedge r) \rightarrow s) \vee \neg r$

(1) draw the parse trees, (2) list all sub-formulas, and (3) use the parse tree to evaluate the formulas when p, r are T and q, s, l are F.

2. **Truth-tables:** Complete the truth-tables for the formulas listed in the above exercise.

3. **Soundness:** Complete the proof of the soundness theorem, i.e., fill in the details of the induction argument in the following cases

$$(\wedge e_1), (\wedge e_2), (\vee i_1), (\vee i_2), (\rightarrow i), (\rightarrow e), (\perp e), (\neg\neg e), (\neg i), \neg(e).$$

4. **Tautology:** Prove that $\phi = p \rightarrow (\neg p \rightarrow q)$ is a tautology, i.e. $\models \phi$. Find a proof for $\vdash \phi$ following step by step the proof of the completeness theorem.

5. **Semantical equivalence:** Which of the following formulas are semantically equivalent to $p \rightarrow (q \vee r)$?

1. $q \vee (\neg p \vee r)$
2. $q \wedge \neg r \rightarrow p$
3. $p \wedge \neg r \rightarrow q$
4. $\neg q \wedge \neg r \rightarrow \neg p$

6. **CNF(2):** Apply the rewriting algorithm to find a CNF for the following formula $\neg(p \rightarrow (\neg(q \wedge (\neg p \rightarrow q))))$.

7. **Horn clauses** Apply HORN algorithm to find whether the following formulas are satisfiable or not.

1. $(p \wedge q \wedge w \rightarrow \perp) \wedge (t \rightarrow \perp) \wedge (r \rightarrow p) \wedge (\top \rightarrow r) \wedge (\top \rightarrow q) \wedge (r \wedge u \rightarrow w) \wedge (u \rightarrow s) \wedge (\top \rightarrow u)$
2. $(\top \rightarrow q) \wedge (\top \rightarrow s) \wedge (w \rightarrow \perp) \wedge (p \wedge q \wedge s \rightarrow \perp) \wedge (v \rightarrow s) \wedge (\top \rightarrow r) \wedge (r \rightarrow p)$