

## Tutorial 3

1. Use the predicates

$A(x, y) :$   $x$  admires  $y$   
 $B(x, y) :$   $x$  attended  $y$   
 $P(x) :$   $x$  is a professor  
 $S(x) :$   $x$  is a student  
 $L(x) :$   $x$  is a lecture

and the function constant

$m :$  Mary

to translate the following into predicate logic:

- (a) Mary admires every professor.
- (b) Some professor admires Mary.
- (c) Mary admires herself.
- (d) No student attended every lecture.
- (e) No lecture was attended by every student.
- (f) No lecture was attended by any student.

2. Consider the following formula, denoted by  $\Phi$ :

$$\neg(\forall x ((\exists y P(x, y, z)) \wedge (\forall z P(x, y, z))))$$

- (a) Draw the parse tree of  $\Phi$ .
- (b) Indicate the free and bound variables in that parse tree.
- (c) List all variables which occur free *and* bound therein.
- (d) Compute  $\Phi[t/x]$ ,  $\Phi[t/y]$ , and  $\Phi[t/z]$ , where  $t$  equals the term  $g(f(g(y, y)), y)$ . Is  $t$  free for  $x$  in  $\Phi$ ? Is  $t$  free for  $y$  in  $\Phi$ ? Is  $t$  free for  $z$  in  $\Phi$ ?

3. Prove the following sequents in predicate logic, using natural deduction rules.

- (a)  $\forall x (P(x) \wedge Q(x)) \vdash \forall x P(x) \wedge \forall x Q(x)$
- (b)  $\exists x P(x) \vee \exists x Q(x) \vdash \exists x (P(x) \vee Q(x))$
- (c)  $\forall x \forall y P(x, y) \vdash \forall u \forall v P(u, v)$
- (d)  $\exists x \forall y P(x, y) \vdash \forall y \exists x P(x, y)$
- (e)  $P(a) \vdash \forall x (x = a \rightarrow P(x))$
- (f)  $\forall x P(x) \rightarrow S \vdash \exists y (P(y) \rightarrow S)$  ( $S$  is a predicate with 0 arguments)