Tutorial 4

1. Consider the formula
$$\Phi \stackrel{def}{=} \forall x \forall y \, Q(g(x,y), g(y,y), z).$$

Obviously, Q is a predicate with three arguments and g is a function with two arguments. Find two models $\mathcal{M}$ and $\mathcal{M}'$ with respective environments l and l' such that $\mathcal{M} \models_l \Phi$, but $\mathcal{M}' \not\models_{l'} \Phi$.

2. Show the semantic entailment

$$\forall x \, P(x) \vee \forall x \, Q(x) \models \forall x \, (P(x) \vee Q(x))$$

3. For each of the formulas of predicate logic below, either find a model which does not satisfy it, or prove it without any premises:

(a) $(\forall x \forall y \, (S(x,y) \to S(y,x))) \to (\forall x \, \neg S(x,x))$

(b) $\exists y \, ((\forall x \, P(x)) \to P(y))$

(c) $(\forall x \, (P(x) \to \exists y \, Q(y))) \to (\forall x \exists y \, (P(x) \to Q(y)))$

(d) $(\forall x \exists y \, (P(x) \to Q(y))) \to (\forall x \, (P(x) \to \exists y \, Q(y)))$

(e) $\forall x \forall y \, (S(x,y) \to (\exists z \, (S(x,z) \wedge S(z,y))))$

(f) $(\forall x \forall y \, (S(x,y) \to (x=y))) \to (\forall z \neg S(z,z))$

(g) $(\forall x \exists y \, (S(x,y) \wedge ((S(x,y) \wedge S(y,x)) \to (x=y)))) \to (\neg \exists z \forall w \, (S(z,w)))$