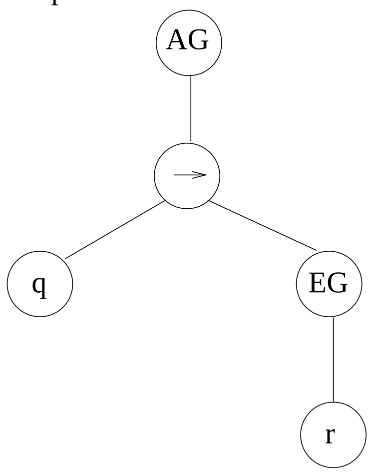
CS3234 - Tutorial 7, Solutions

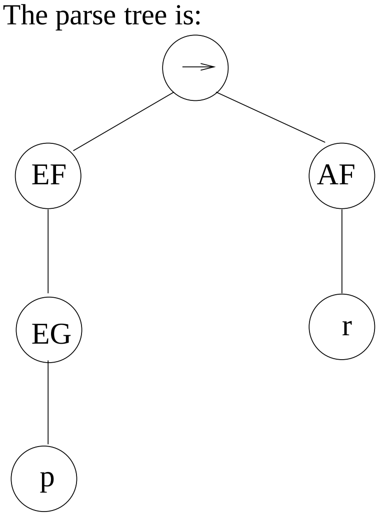
1.

1. AG$(q \rightarrow$ EG $r)$. It's a CTL formula.
The parse tree is:



The subformulas are: q, r, EG r, $q \rightarrow$ EG r, initial formula.

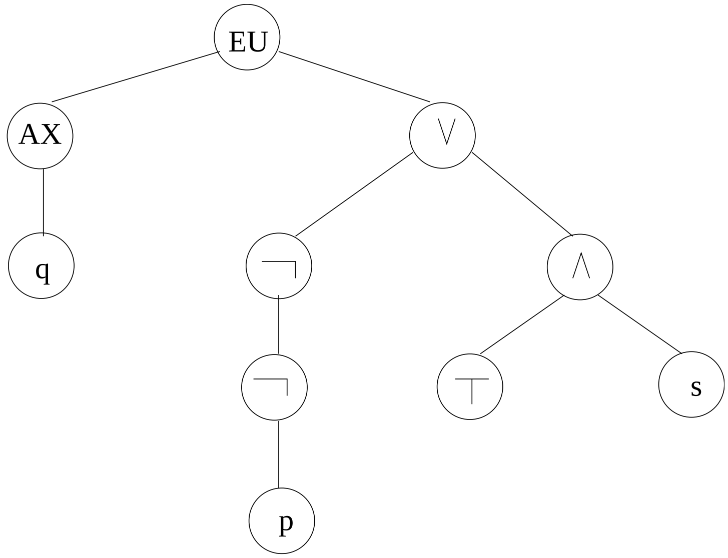
2. EF EG $p \rightarrow$ AF r. It's a CTL formula.
The parse tree is:



The subformulas are: p, EG p, r, EF(EG p), AF r, initial formula.

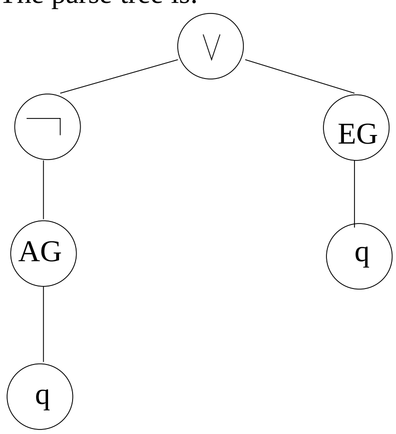
3. AF$[(r$ U $q) \land (p$ U $q)]$. It's not a CTL formula because there is no path quantifier for the until ("U") operators.

4. E$[($AX $q)$ U $(\neg(\neg p) \lor (\top \land s))]$. It's a CTL formula.
The parse tree is:



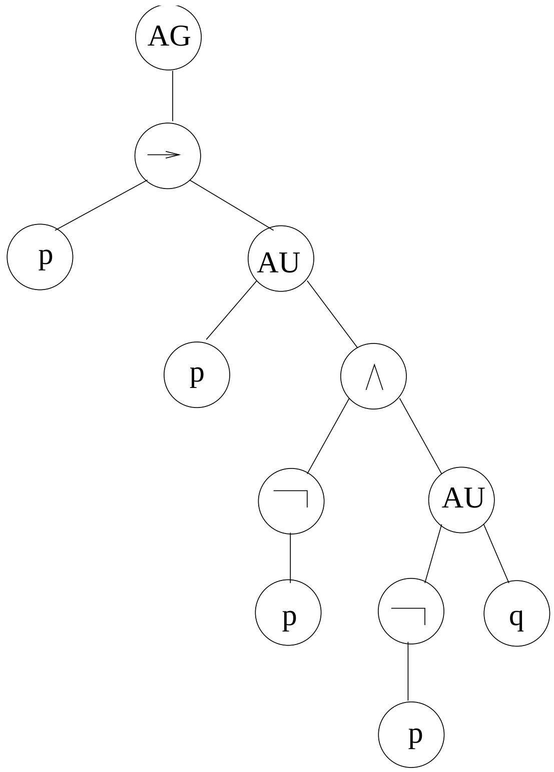
The subformulas are: p, $\neg p$, $\top$, s, q, $\neg(\neg p)$, $\top \wedge s$, AX q, $(\neg(\neg p)) \vee (\top \wedge s)$, initial formula.

5. $\neg(\mathrm{AG}\, q) \vee (\mathrm{EG}\, q)$. It's a CTL formula.
The parse tree is:



The subformulas are: q, AG q, $\neg(\mathrm{AG}\, q)$, EG q, initial formula.

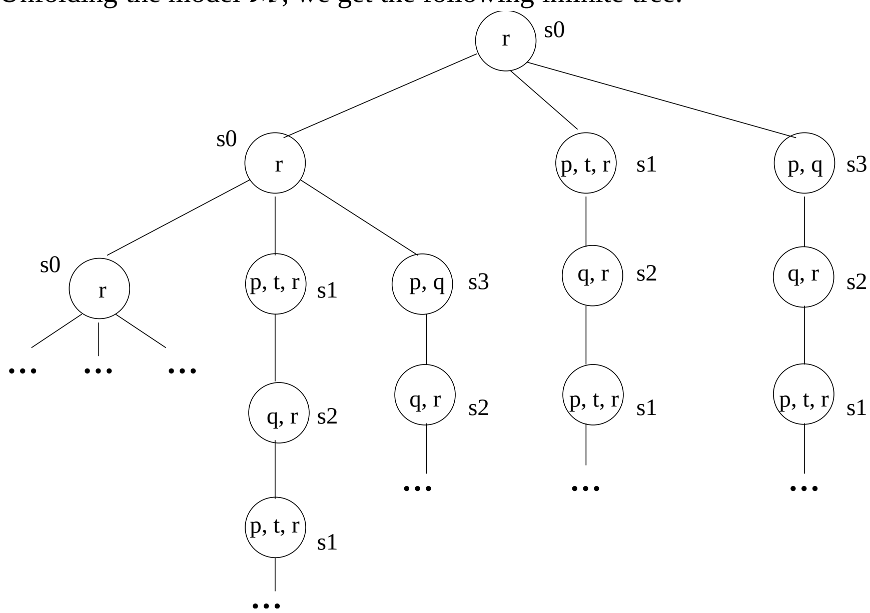
6. $\mathrm{AG}(p \rightarrow \mathrm{A}[p \mathbin{\mathrm{U}} (\neg p \wedge \mathrm{A}[\neg p \mathbin{\mathrm{U}} q])])$. it's a CTL formula.
The parse tree is:



The subformulas are: p, $\neg p$, q, $A[(\neg p) \mathrm{U}\, q]$, $(\neg p) \vee (A[(\neg p) \mathrm{U}\, q])$, $A[p\, \mathrm{U}\, ((\neg p) \vee (A[(\neg p) \mathrm{U}\, q]))]$, $p \rightarrow A[p\, \mathrm{U}\, ((\neg p) \vee (A[(\neg p) \mathrm{U}\, q]))]$, initial formula.

2.

1. Unfolding the model $\mathcal{M}$, we get the following infinite tree:



2. Checking that $\mathcal{M}, s_0 \models \phi$ for the following ϕ's:

1. $\neg p \rightarrow r$. YES, r is true in s_0.
2. AF t. NO, because there is an infinite path along which t is always false ($s_0 \rightarrow s_0 \rightarrow s_0 \rightarrow \ldots$)
3. $\neg$EG r. NO, because there is an infinite path along which r is always true ($s_0 \rightarrow s_0 \rightarrow s_0 \rightarrow \ldots$), so EG r is true.
4. E$[t$ U $q]$. NO, because in s_0, neither t nor q are true.
5. AF q. NO, because there is an infinite path along which q is always false.
6. EF q. YES, consider the path $s_0 \rightarrow s_3 \rightarrow s_2 \rightarrow s_1 \rightarrow \ldots$, where q is true in s_3.
7. EG r. YES, consider the path $s_0 \rightarrow s_0 \rightarrow s_0 \rightarrow \ldots$.
8. AG$(r \vee q)$. YES, because every state in the graph has r or q true.

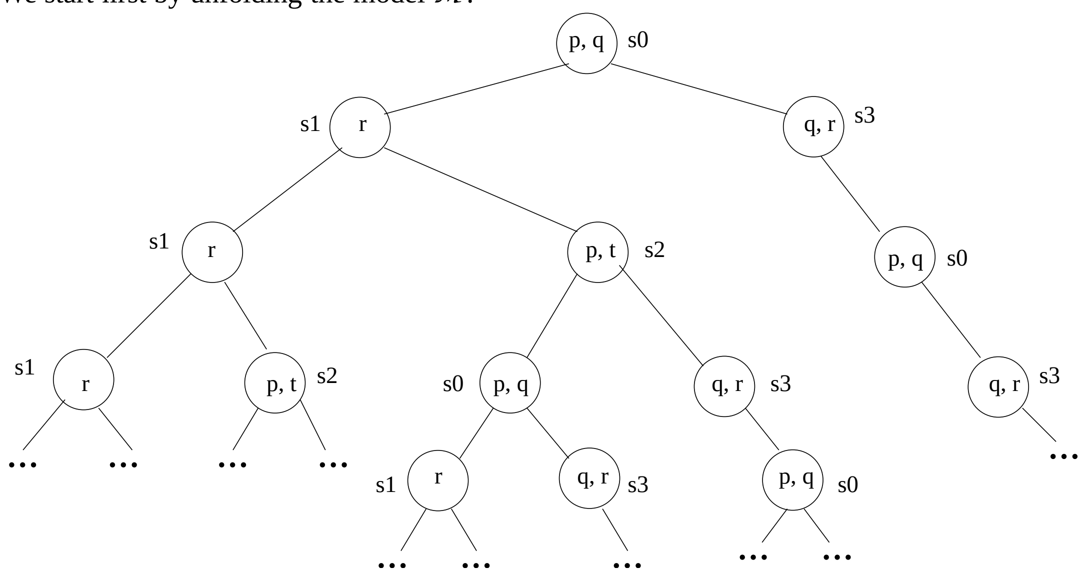
3. Checking that $\mathcal{M}, s_2 \models \phi$, for the followings:

1. $\neg p \rightarrow r$. YES, r is true in s_2.
2. AF t. YES, because unfolding from s_2, there is one single path along which we find t true ($s_2 \rightarrow s_1 \rightarrow s_2 \rightarrow s_1 \rightarrow \ldots$)
3. $\neg$EG r. NO, because r is true in every state along the path $s_2 \rightarrow s_1 \rightarrow s_2 \rightarrow s_1 \rightarrow \ldots$

4. E[t U q]. YES, because q is true in s_2.
5. AF q. YES, because q is true in s_2
6. EF q. YES, because q is true in s_2
7. EG r. YES, because r is true in every state along the path $s_2 \to s_1 \to s_2 \to s_1 \to \ldots$.
8. AG$(r \vee q)$. YES, because every state along the path $s_2 \to s_1 \to s_2 \to s_1 \to \ldots$ has r or q true.

3.

- We start first by unfolding the model $\mathcal{M}$:



1. Checking that $\mathcal{M}, s_0 \models \phi$, for the followings:

1. AF q. YES, because q is true in s_0
2. AG(EF$(p \vee r)$). YES, because every state in the graph has p or r true.
3. EX(EX r). YES, because along the path $s_0 \to s_1 \to s_1 \to s_1 \to \ldots$ we find the second s_1 state where r is true.
4. AG(AF q). NO, because the property AF q should hold in every state (because of AG), and we have an infinite path $s_1 \to s_1 \to s_1 \to \ldots$ along which q is false.

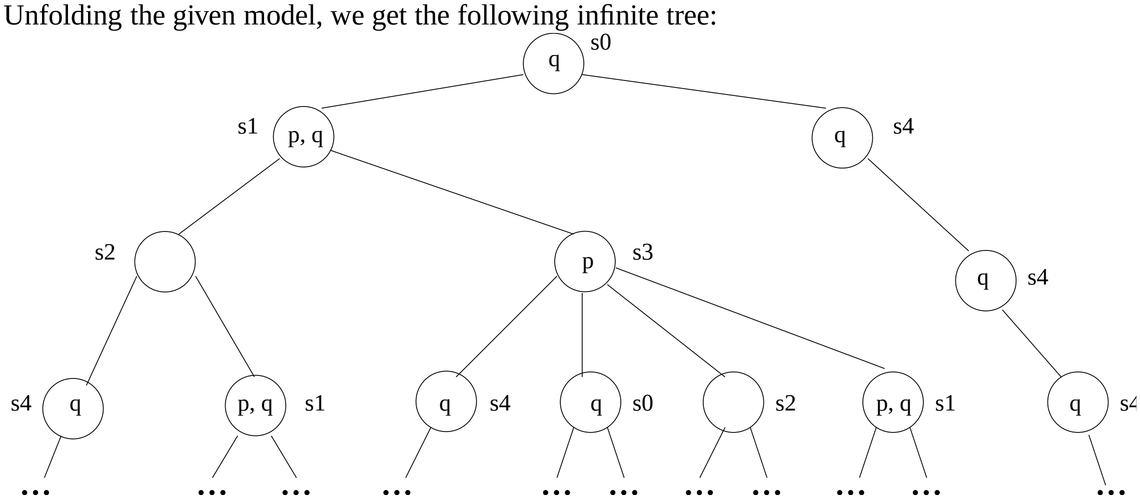
2. Checking that $\mathcal{M}, s_2 \models \phi$, for the followings:

1. AF q. YES, because from s_2 you can advance to s_0 or s_3 only, and in both these states you have q true.
2. AG(EF $(p \vee r)$). YES, because every state in the graph has p or r true.

3. EX(EX r). YES, because along the path $s_2 \to s_0 \to s_1 \to s_1 \to \ldots$ we find the s_1 state (first one) where r is true.
4. AG(AF q). NO, because the property AF q should hold in every state (because of AG), and we have an infinite path $s_1 \to s_1 \to s_1 \to \ldots$ along which q is false.

4.

- Unfolding the given model, we get the following infinite tree:



- Checking the truth value of the formula $\phi = \mathrm{AG}(p \to \mathrm{A}[p\,\mathrm{U}\,(\neg p \wedge \mathrm{A}[\neg p\,\mathrm{U}\,q])])$ for the model and each state s_0, s_1, s_2, s_3, s_4:
  - $\mathcal{M}, s_0 \models \phi$. NO, consider the following path $s_0 \to s_1 \to s_3 \to s_1 \to s_3 \to \ldots$
  - $\mathcal{M}, s_1 \models \phi$. NO, consider the following path $s_1 \to s_3 \to s_1 \to s_3 \to \ldots$
  - $\mathcal{M}, s_2 \models \phi$. NO, consider the following path $s_2 \to s_1 \to s_3 \to s_1 \to s_3 \to \ldots$
  - $\mathcal{M}, s_3 \models \phi$. NO, consider the following path $s_3 \to s_1 \to s_3 \to s_1 \to \ldots$
  - $\mathcal{M}, s_4 \models \phi$. YES, p is false in s_4

5.

1. $\mathrm{A}[(\neg s \wedge \neg t)\ \mathrm{U}\ (\neg s \wedge \neg t \wedge p \wedge (\mathrm{EF}\, s \vee \mathrm{EF}\, t)]$

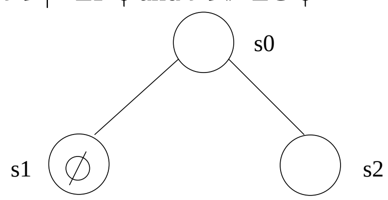
2. $\mathrm{AG}(p \to \mathrm{AG}\neg q)$

3. $\mathrm{AG}((r \to \mathrm{A}[\neg p\,\mathrm{U}\,q]) \wedge (q \to \mathrm{A}[\neg p\,\mathrm{U}\,r]))$

4. ”**on every path** *transitions to states satisfying p occur at most twice*”:
$(\neg p \wedge \mathrm{AX\,A}[\neg p\ \mathrm{U}\ (p \wedge \mathrm{AX\,A}[\neg p\ \mathrm{U}\ (p \wedge \mathrm{AG}\neg p)])]) \vee$
$(\neg p \wedge \mathrm{AXA}[\neg p\ \mathrm{U}\ (p \wedge \mathrm{AG}\neg p)]) \vee \mathrm{AG}\neg p$

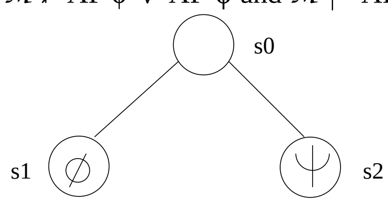
6.

1. $\mathcal{M} \models \mathrm{EF}\,\phi$ and $\mathcal{M} \not\models \mathrm{EG}\,\phi$



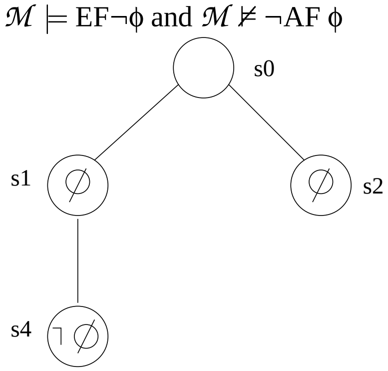
2. $\mathrm{EF}\,\phi \vee \mathrm{EF}\,\psi \equiv \mathrm{EF}(\phi \vee \psi)$

3. $\mathcal{M} \not\models \mathrm{AF}\,\phi \vee \mathrm{AF}\,\psi$ and $\mathcal{M} \models \mathrm{AF}(\phi \vee \psi)$

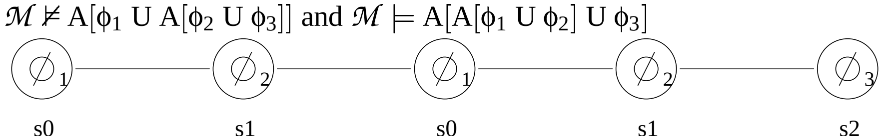


4. $\mathrm{AF}\neg\phi \equiv \neg\mathrm{EG}\,\phi$

5. $\mathcal{M} \models \mathrm{EF}\neg\phi$ and $\mathcal{M} \not\models \neg\mathrm{AF}\,\phi$

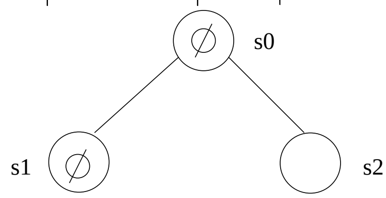


6. $\mathcal{M} \not\models \mathrm{A}[\phi_1\ \mathrm{U}\ \mathrm{A}[\phi_2\ \mathrm{U}\ \phi_3]]$ and $\mathcal{M} \models \mathrm{A}[\mathrm{A}[\phi_1\ \mathrm{U}\ \phi_2]\ \mathrm{U}\ \phi_3]$



7. $\top \equiv \text{AG } \phi \rightarrow \text{EG } \phi$

8. $\mathcal{M} \models \top$ and $\mathcal{M} \models \text{EG } \phi$ but $\mathcal{M} \not\models \text{AG } \phi$, so $\mathcal{M} \not\vdash \text{EG } \phi \;\; \rightarrow \;\; \text{AG } \phi$



7.

1. Let us consider a model $\mathcal{M}$ for $\neg\text{AF } \phi$
We have the following equivalences:

$\mathcal{M}, s_0 \models \neg\text{AF } \phi$
iff
is not true that for all paths $s_0 \rightarrow s_1 \rightarrow s_2 \ldots$ exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \phi$
iff
there is a path $s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots$ where is not true that there exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \phi$
iff
there is a path $s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots$ where for all $i \in \mathbb{N}$ is not true that $\mathcal{M}, s_i \models \phi$
iff
there is a path $s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots$ where for all $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \neg\phi$
iff
$\mathcal{M}, s_0 \models \text{EG}\neg\phi$

2. Let us consider a model $\mathcal{M}$ for $\neg\text{EF } \phi$
We have the following equivalences:

$\mathcal{M}, s_0 \models \neg\text{EF } \phi$
iff
is not true that exists a path $s_0 \rightarrow s_1 \rightarrow s_2 \ldots$ such that exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \phi$
iff
for all paths $s_0 \rightarrow s_1 \rightarrow s_2 \ldots$ is not true that exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \phi$
iff
for all paths $s_0 \rightarrow s_1 \rightarrow s_2 \ldots$ and for all $i \in \mathbb{N}$ is not true that $\mathcal{M}, s_i \models \phi$
iff
for all paths $s_0 \rightarrow s_1 \rightarrow s_2 \ldots$ and for all $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \neg\phi$
iff
$\mathcal{M}, s_0 \models AG\neg\phi$

3. Let us consider a model $\mathcal{M}$ for $\neg\text{AX } \phi$
We have the following equivalences:

$\mathcal{M}, s_0 \models \neg \mathrm{AX}\, \phi$
iff
is not true that for all paths $s_0 \to s_1 \to s_2 \ldots$, $\mathcal{M}, s_1 \models \phi$
iff
exists a path $s_0 \to s_1 \to s_2 \ldots$ such that is not true $\mathcal{M}, s_1 \models \phi$
iff
exists a path $s_0 \to s_1 \to s_2 \ldots$ such that $\mathcal{M}, s_1 \models \neg\phi$
iff
$\mathcal{M}, s_0 \models \mathrm{EX}\neg\phi$

4. Let us consider a model $\mathcal{M}$ for $\mathrm{A}[\top\ \mathrm{U}\ \phi]$
We have the following equivalences:

$\mathcal{M}, s_0 \models \mathrm{A}[\top\ \mathrm{U}\ \phi]$
iff
for all paths $s_0 \to s_1 \to s_2 \ldots$, exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \phi$ and for all $j < i$, $\mathcal{M}, s_j \models \top$
iff
for all paths $s_0 \to s_1 \to s_2 \ldots$, exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \phi$
iff
$\mathcal{M}, s_0 \models \mathrm{AF}\phi$

5. Let us consider a model $\mathcal{M}$ for $\mathrm{E}[\top \mathrm{U} \phi]$
We have the following equivalences:

$\mathcal{M}, s_0 \models \mathrm{E}[\top\ \mathrm{U}\ \phi]$
iff
exists a path $s_0 \to s_1 \to s_2 \ldots$ such that exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \phi$ and for all $j < i$, $\mathcal{M}, s_j \models \top$
iff
exists a path $s_0 \to s_1 \to s_2 \ldots$ such that exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \phi$
iff
$\mathcal{M}, s_0 \models \mathrm{EF}\, \phi$

6. Let us consider a model $\mathcal{M}$ for $\neg(\mathrm{E}[\neg\psi\ \mathrm{U}\ (\neg\phi \wedge \neg\psi)] \ \vee\ \mathrm{EG}\neg\psi)$
We have the following equivalences:

$\mathcal{M}, s_0 \models \neg(\mathrm{E}[\neg\psi\ \mathrm{U}\ (\neg\phi \wedge \neg\psi)] \ \vee\ \mathrm{EG}\neg\psi)$
iff
is not true that

> exists a path $s_0 \to s_1 \to s_2 \ldots$ such that exists $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \neg\phi \wedge \neg\psi$ and for all $j < i$, $\mathcal{M}, s_j \models \neg\psi$

or

> exists a path $s_0 \to s_1 \to s_2 \ldots$ such that for all $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \neg\psi$

iff

is not true that exists a path $s_0 \to s_1 \to s_2 \ldots$ such that exists $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \neg\phi \wedge \neg\psi$ and for all $j < i$, $\mathcal{M}, s_j \models \neg\psi$

and

is not true that exists a path $s_0 \to s_1 \to s_2 \ldots$ such that for all $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \neg\psi$

iff

for all paths $s_0 \to s_1 \to s_2 \ldots$ is not true that exists $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \neg\phi \wedge \neg\psi$ and for all $j < i$, $\mathcal{M}, s_j \models \neg\psi$

and

for all paths $s_0 \to s_1 \to s_2 \ldots$ is not that for all $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \neg\psi$

iff

for all paths $s_0 \to s_1 \to s_2 \ldots$ for all $i \in \mathbb{N}$ is not true that $\mathcal{M}, s_i \models \neg\phi \wedge \neg\psi$ and for all $j < i$, $\mathcal{M}, s_j \models \neg\psi$

and

for all paths $s_0 \to s_1 \to s_2 \ldots$ exists $i \in \mathbb{N}$ such that is not true $\mathcal{M}, s_i \models \neg\psi$

iff

for all paths $s_0 \to s_1 \to s_2 \ldots$ for all $i \in \mathbb{N}$ is not true that $\mathcal{M}, s_i \models \neg\phi$ and $\mathcal{M}, s_i \models \neg\psi$ and for all $j < i$, $\mathcal{M}, s_j \models \neg\psi$

and

for all paths $s_0 \to s_1 \to s_2 \ldots$ exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \psi$

iff

for all paths $s_0 \to s_1 \to s_2 \ldots$ for all $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \phi$ or $\mathcal{M}, s_i \models \psi$ or is not true that for all $j < i$, $\mathcal{M}, s_j \models \neg\psi$

and

for all paths $s_0 \to s_1 \to s_2 \ldots$ exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \psi$

iff

for all paths $s_0 \to s_1 \to s_2 \ldots$ for all $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \phi$ or $\mathcal{M}, s_i \models \psi$ or exists $j < i$ such that is not true $\mathcal{M}, s_j \models \neg\psi$

and

for all paths $s_0 \to s_1 \to s_2 \ldots$ exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \psi$

iff

for all paths $s_0 \to s_1 \to s_2 \ldots$ for all $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \phi$ or $\mathcal{M}, s_i \models \psi$ or exists $j < i$ such that $\mathcal{M}, s_j \models \psi$

and

for all paths $s_0 \to s_1 \to s_2 \ldots$ exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \psi$

iff

for all paths $s_0 \to s_1 \to s_2 \ldots$ for all $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \phi \vee \psi$ or exists $j < i$ such that $\mathcal{M}, s_j \models \psi$

and

for all paths $s_0 \to s_1 \to s_2 \ldots$ exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \psi$

iff

for all paths $s_0 \to s_1 \to s_2 \ldots$ for all $i \in \mathbb{N}$, $\mathcal{M}, s_i \models \phi \vee \psi$ or exists $j < i$ such that $\mathcal{M}, s_j \models \psi$

and

for all paths $s_0 \to s_1 \to s_2 \ldots$ exists $i_0 \in \mathbb{N}$ such that $\mathcal{M}, s_{i_0} \models \psi$ and for all $j < i_0$, $\mathcal{M}, s_j \models \neg\psi$ (i_0 is the smallest i with the property $\mathcal{M}, s_i \models \psi$)

iff

for all paths $s_0 \to s_1 \to s_2 \ldots$ exists $i_0 \in \mathbb{N}$ such that $\mathcal{M}, s_{i_0} \models \psi$ and for all $j < i_0$ $\mathcal{M}, s_j \models \neg\psi$ and $\mathcal{M}, s_j \models \phi \vee \psi$

iff
for all paths $s_0 \to s_1 \to s_2 \ldots$ exists $i_0 \in \mathbb{N}$ such that $\mathcal{M}, s_{i_0} \models \psi$ and for all $j < i_0$ $\mathcal{M}, s_j \models \neg\psi \wedge (\phi \vee \psi)$
iff
for all paths $s_0 \to s_1 \to s_2 \ldots$ exists $i_0 \in \mathbb{N}$ such that $\mathcal{M}, s_{i_0} \models \psi$ and for all $j < i_0$ $\mathcal{M}, s_j \models \neg\psi \wedge \phi$
iff
for all paths $s_0 \to s_1 \to s_2 \ldots$ exists $i \in \mathbb{N}$ such that $\mathcal{M}, s_i \models \psi$ and for all $j < i$ $\mathcal{M}, s_j \models \phi$
iff
$\mathcal{M}, s_0 \models \mathrm{A}[\phi\ \mathrm{U}\ \psi]$