National University of Singapore
School of Computing
CS3234 — Logic and Formal Systems
Semester I, 2004/2005

Tutorial 6

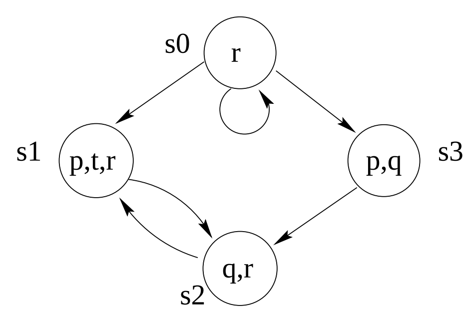
1. **Syntax**: For each of the following strings, state if it is a well-formed CTL formula or not; for the well-formed formulas draw the parse tree and list all subformulas.

1	$\text{AG}(q \to \text{EG } r)$	2	$\text{EF EG } p \to \text{AF } r$
3	$\text{AF}[(r \text{ U } q) \wedge (p \text{ U } r)]$	4	$\text{E}[(\text{AX } q) \text{ U } (\neg(\neg p) \vee (\top \wedge s))]$
5	$\neg(\text{AG } q) \vee (\text{EG } q)$	6	$\text{AG}(p \to \text{A}[p \text{ U } (\neg p \wedge \text{A}[\neg p \text{ U } q])])$

2. **Semantics-1**:

   Consider the system $\mathcal{M}$ in the figure.

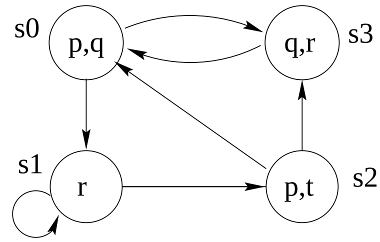
   1. Unfold it to get an infinite tree;
   2. Check $\mathcal{M}, s_0 \models \phi$ for the following formulas:
      1. $\neg p \to r$
      2. $\text{AF } t$
      3. $\neg\text{EG } r$
      4. $\text{E}(t \text{ U } q)$
      5. $\text{AF } q$
      6. $\text{EF } q$
      7. $\text{EG } r$
      8. $\text{AG}(r \vee q)$
   3. Repeat 2, but for the state $s2$



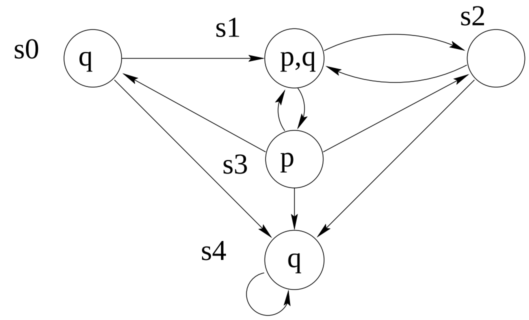
3. Semantics-2:

Consider the system $\mathcal{M}$ in the figure.

1. Check $\mathcal{M}, s_0 \models \phi$ for the following formulas:
   1. $\text{AF } q$
   2. $\text{AG}(\text{EF}(p \vee r)$
   3. $\text{EX}(\text{EX } r)$
   4. $\text{AG}(\text{AF } q)$
2. Repeat 1, but for state $s2$



4. Semantics-3:



Find the truth value of the formula $\phi = \text{AG}(p \rightarrow \text{A}[p \text{ U } (\neg p \wedge \text{A}[\neg p \text{ U } q])])$ for the model described in the figure and each state $s0, s1, s2, s3$ and $s4$.

5. Specification: Write CTL formulas for:

1. "p precedes s and t on all computation paths"
2. "always after p, q is never true"
3. "between the events q and r, p is never true"
4. "transitions to states satisfying p occur at most twice"

6. Equivalent formulas-1: Which of the following pairs of CTL formulas are equivalent? (When not, describe a model for one which is not a model for the other.)

1	$\text{EF } \phi$ and $\text{EG } \phi$	2	$\text{EF } \phi \vee \text{EF } \psi$ and $\text{EF}(\phi \vee \psi)$
3	$\text{AF } \phi \vee \text{AF } \psi$ and $\text{AF}(\phi \vee \psi)$	4	$\text{AF}\neg\phi$ and $\neg\text{EG } \phi$
5	$\text{EF}\neg\phi$ and $\neg\text{AF } \phi$	6	$\text{A}[\phi_1 \text{ U } \text{A}[\phi_2 \text{ U } \phi_3]]$ and $\text{A}[\text{A}[\phi_1 \text{ U } \phi_2] \text{ U } \phi_3]$
7	$\top$ and $\text{AG } \phi \rightarrow \text{EG } \phi$	8	$\top$ and $\text{EG } \phi \rightarrow \text{AG } \phi$

7. Equivalent formulas-2: Prove that the following equivalences hold.

1	$\neg\text{AF } \phi \equiv \text{EG } \neg\phi$	2	$\neg\text{EF } \phi \equiv \text{AG } \neg\phi$
3	$\neg\text{AX } \phi \equiv \text{EX } \neg\phi$	4	$\text{AF } \phi \equiv \text{A}[\top \text{ U } \phi]$
5	$\text{EF } \phi \equiv \text{E}[\top \text{ U } \phi]$	6	$\text{A}[\phi \text{ U } \psi] \equiv \neg(\text{E}[\neg\psi \text{ U } (\neg\phi \wedge \neg\psi)] \vee \text{EG } \neg\psi)$