National University of Singapore
School of Computing
CS3234 — Logic and Formal Systems
Semester I, 2004/2005

Tutorial 8

1. **From SMV to graphs**: Draw the transition system described by the ABP (Alternating Bit Protocol) SMV program. (Only the reachable states are to be included.)

2. **SMV runnings**: ABP works fine starting from an appropriate initial state. Change the initial state in module `main` in all possible ways (32 cases), run the program, and explain the results.

3. NNF **for LTL and CTL***: Extend the algorithm NNF for computing negation normal form of propositional logic formulas to LTL and CTL*.

4. **CTL***: Provide models to show that the following pairs contain nonequivalent CTL* formulas:

   1. $\mathrm{AFG}\ p$ and $\mathrm{AF}\ \mathrm{AG}\ p$
   2. $\mathrm{AGF}\ p$ and $\mathrm{AG}\ \mathrm{EF}\ p$
   3. $\mathrm{A}[(p\ \mathrm{U}\ r) \vee (q\ \mathrm{U}\ r)]$ and $\mathrm{A}[(p \vee q)\ \mathrm{U}\ r]$
   4. $\mathrm{A}[\mathrm{X}\ p \vee \mathrm{XX}\ p]$ and $\mathrm{AX}\ p \vee \mathrm{AX}\ \mathrm{AX}\ p$
   5. $\mathrm{E}[\mathrm{GF}\ p]$ and $\mathrm{EG}\ \mathrm{EF}\ p$

5. **Boolean path combinations in CTL**: Find (plain) CTL formulas to describe the following boolean combination (similar to the approach in notices, Slide 9.22)

   1. $\mathrm{E}[\mathrm{F}\ p \wedge (q\ \mathrm{U}\ r)]$
   2. $\mathrm{E}[\mathrm{F}\ p \wedge \mathrm{G}\ q]$

   Use them and the ones provided in Slide 9.22 to translate in plain CTL the following extended CTL formulas:

   1. $\mathrm{E}[(p\ \mathrm{U}\ q) \wedge \mathrm{F}\ p]$
   2. $\mathrm{A}[(p\ \mathrm{U}\ q) \wedge \mathrm{G}\ p]$

3. $\text{A}[\text{F}\ p \rightarrow \text{F}\ q]$

6. Fixed points (1): Consider the following functions:

$$H_1, H_2, H_3 : \mathcal{P}(\{1,2,3,4,5,6,7,8,9,10\}) \rightarrow \mathcal{P}(\{1,2,3,4,5,6,7,8,9,10\})$$

defined by

$$H_1(Y) = Y - \{1,4,7\}$$
$$H_2(Y) = \{2,5,9\} - Y$$
$$H_3(Y) = \{1,2,3,4,5\} \cap (\{2,4,8\} \cup Y)$$

1. Which of these functions are monotone and which are not monotone (justify).
2. Compute the least and the greatest fixed points of H_3.
3. Has H_2 any fixed point?

7. Fixed points (2): (a) For each of the following fixed point identities

6 $\text{AG}\ \phi \equiv \phi \wedge \text{AX AG}\ \phi$

7 $\text{EG}\ \phi \equiv \phi \wedge \text{EX EG}\ \phi$

8 $\text{AF}\ \phi \equiv \phi \vee \text{AX AF}\ \phi$

9 $\text{EF}\ \phi \equiv \phi \vee \text{EX EF}\ \phi$

10 $\text{A}[\phi\ \text{U}\ \psi] \equiv \psi \vee (\phi \wedge \text{AX A}[\phi\ \text{U}\ \psi])$

11 $\text{E}[\phi\ \text{U}\ \psi] \equiv \psi \vee (\phi \wedge \text{EX E}[\phi\ \text{U}\ \psi])$

specify if the fixed point is the least fixed point, the greatest fixed point, or none of them for the associated function.

(b) What is the meaning of the temporal operator '??' defined by the following relation

$$??(X,Y) = \nu Z.\ X \cap (Y \cup \text{AX}\ Z)$$