School of Computing
National University of Singapore
CS5240 Theoretical Foundations in Multimedia

Exercise 1

Working with Matrix Elements

Objectives

- This exercise lets you practice working with matrix elements.
- You should learn to work out the answers **yourself** without referring to Google, Wikipedia, etc., or consulting others.
- Work out the answers using the simplest, cleanest and most concise method.

Exercise Questions

Let's denote the (i, j)-th elements of matrix $\mathbf{A}$ and $\mathbf{B}$ as $[\mathbf{A}]_{ij}$ and $[\mathbf{B}]_{ij}$, respectively. Then, the (i, j)-th element of the matrix product $\mathbf{AB}$ is

$$[\mathbf{AB}]_{ij} = \sum_k [\mathbf{A}]_{ik}[\mathbf{B}]_{kj}.$$

Then, matrix transpose means $[\mathbf{A}^\top]_{ij} = [\mathbf{A}]_{ji}$.

Work on the following exercises. The first exercise is partially worked out for you as an illustrative example.

(a) Show that $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$.
The (i, j)-th element of the left hand side is

$$[\mathbf{A}(\mathbf{B} + \mathbf{C})]_{ij} = \sum_k [\mathbf{A}]_{ik}[\mathbf{B} + \mathbf{C}]_{kj} = \cdots$$

(b) Show that $(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$. Note that $\mathbf{AB} \neq \mathbf{BA}$.

(c) Show that $(\mathbf{A} + \mathbf{B})^\top = \mathbf{A}^\top + \mathbf{B}^\top$.

(d) Show that $(\mathbf{AB})^\top = \mathbf{B}^\top\mathbf{A}^\top$.