Abstract

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An abelian group admits an order iff it is torsion-free, we consider degrees of orders on computable torsion-free abelian groups. For a computable torsion-free abelian group $\mathcal{G}$, Solomon (2003) showed that if $\mathcal{G}$ has rank 1, then it has exactly two computable orders; if $\mathcal{G}$ has finite rank ≥ 2, then it has orders of all Turing degrees; and if $\mathcal{G}$ has infinite rank, then it has orders of all degrees $\geq \mathbf{0}'$. Motivated by the question whether the set of all degrees of orders on a computable torsion-free abelian group is closed upwards, Kach, Lange and Solomon (2013) constructed a computable torsion-free abelian group $\mathcal{G}$ of infinite rank with exactly two computable orders and a noncomputable, computably enumerable (c.e. for short) set C such that every C-computable order on $\mathcal{G}$ is computable, so this $\mathcal{G}$ has no orders in $deg(C) > \mathbf{0}$, but has orders in $\mathbf{0}$, a negative answer is provided for above question.

One proposed research topic is to study the computational complexity of above c.e. set C from the standpoint of classical computability theory, we will present a recent work on this topic from the viewpoint of high/low hierarchy. This is a joint work with my supervisor Wu Guohua.