A RAMSEY THEOREM FOR METRIC SPACES

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We use the following variation of the standard “Hungarian” arrow notation which takes into account additional structure:

**Definition 1.** Let $\mathcal{K}$ be a class of structures and $\kappa, \lambda, \nu$ be cardinals. The arrow

$$\kappa \rightarrow_{\mathcal{K}} (\lambda)^1_{\nu},$$

is shorthand for the statement that for every $K \in \mathcal{K}$ of size $\lambda$ there is a $Y \in \mathcal{K}$ of size $\kappa$ such that for any partition of $Y$ into $\mu$-many pieces one of the pieces contains an isomorphic copy of $K$.

In the talk we will investigate this arrow in the case where $\mathcal{K}$ is the class of bounded metric spaces with “isomorphic copies” being scaled copies. We extend previous work on these questions (e.g. [1], [2], [3], [4]). In particular, W. Weiss shows in [4] that there is a limit to what one can prove:

**Theorem 1 (Weiss).** Assume that there are no measurable cardinals. If $X$ is a topological space then there is a coloring of $X$ by two colours such that $X$ doesn’t contain a monochromatic homeomorphic copy of the Cantor set.

It follows that in the class of metric spaces there are no positive results if $\kappa > \omega$. However the case $\kappa = \omega$ is not ruled out and we prove a positive theorem*: Let $\mathcal{M}$ be the class of bounded metric spaces with “$X$ contains an isomorphic copy of $Y$” being “$X$ contains a subspace which is a scaled copy of $Y$”. ($K$ is a scaled copy of $Y$ if there is a bijection $f : K \rightarrow Y$ onto $Y$ and a scaling factor $c \in \mathbb{R}^+$ such that $d_K(x, y) = c \cdot d_Y(f(x), f(y))$).

**Theorem 2.**

$$2^\omega \rightarrow_{\mathcal{M}} (\omega)^1_{\omega}.$$

Time permitting we will also prove a version for ultrametric spaces where the size of the universal space (i.e. the cardinal on the left of the arrow) is $\aleph_1$. However, we must weaken the conclusion somewhat:

**Theorem 3.** There is a rational ultrametric $(X, \rho)$ of size $\aleph_1$ such that for every coloring of $X$ by countably many colors, $X$ contains a monochromatic isometric copy of every finite rational ultrametric space.
REFERENCES


