A RAMSEY THEOREM FOR METRIC SPACES

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We use the following variation of the standard "Hungarian" arrow notation which takes into account additional structure:

Definition 1. Let $\mathcal{K}$ be a class of structures and κ, λ, ν be cardinals. The arrow

$$\kappa \to_{\mathcal{K}} (\lambda)^1_\mu,$$

is shorthand for the statement that for every $K \in \mathcal{K}$ of size λ there is a $Y \in \mathcal{K}$ of size κ such that for any partition of Y into μ-many pieces one of the pieces contains an isomorphic copy of K.

In the talk we will investigate this arrow in the case where $\mathcal{K}$ is the class of bounded metric spaces with "isomorphic copies" being *scaled copies*. We extend previous work on these questions (e.g. [1], [2], [3], [4]). In particular, W. Weiss shows in [4] that there is a limit to what one can prove:

Theorem 1 (Weiss)**.** Assume that there are no measurable cardinals. If X is a topological space then there is a coloring of X by two colours such that X doesn't contain a monochromatic homeomorphic copy of the Cantor set.

It follows that in the class of metric spaces there are no positive results if $\kappa > \omega$. However the case $\kappa = \omega$ is not ruled out and we prove a positive theorem*: Let $\mathcal{M}$ be the class of bounded metric spaces with "X contains an isomorphic copy of Y" being "X contains a subspace which is a scaled copy of Y". (K is a scaled copy of Y if there is a bijection $f : K \to Y$ onto Y and a scaling factor $c \in \mathbb{R}^+$ such that $d_K(x, y) = c \cdot d_Y(f(x), f(y))$.)

Theorem 2.

$$2^\omega \to_{\mathcal{M}} (\omega)^1_\omega.$$

Time permitting we will also prove a version for ultrametric spaces where the size of the universal space (i.e. the cardinal on the left of the arrow) is $\aleph_1$. However, we must weaken the conclusion somewhat:

Theorem 3. There is a rational ultrametric (X, ρ) of size $\aleph_1$ such that for every coloring of X by countably many colors, X contains a monochromatic isometric copy of every finite rational ultrametric space.

References

[1] Péter Komjáth, *Ramsey-type results for metric spaces*, Journal of Combinatorial Theory, Series A **45** (1987), no. 2, 323–328.

[2] Jaroslav Neetil, *Ramsey classes of topological and metric spaces*, Annals of Pure and Applied Logic **143** (2006), no. 13, 147 – 154, International Meeting on Logic, Algebra and Geometry.

[3] W. Weiss P. Komjáth, *Partitioning topological spaces into countably many pieces*, Proceedings of the American Mathematical Society **101** (1987), no. 4, 767–770.

[4] William Weiss, *Mathematics of ramsey theory*, ch. Partitioning Topological Spaces, pp. 154–171, Springer Berlin Heidelberg, Berlin, Heidelberg, 1990.