Faculty of Science
Department of Mathematics

MA4198 PROJECT PROPOSAL (PROJECT CUM SEMINAR GROUP)

SUPERVISOR'S INFO

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TITLE

The influence of the power of machine models on the complexity of computational problems

BRIEF DESCRIPTION OF PROJECT

There are various machine models which are all Turing complete, that is, lead to the same class of computable or recursive functions. However, for computational problems of low complexity, they give different answer. For example, addition and subtraction of integers (in binary or decimal coding) is in polynomial time for Turing machines while it takes exponential time on counter machines; these can do only the following: increment counters, decrement counters and compare counters with 0, that is do in the program the statement "If Counter5 > 0 Then do Action1 else do Action 2". Floyd and Knuth fixed this by considering addition machines which can compare (<,=,>) and add and subtract integer registers in unit time and therefore also have a much more powerful class of linear time operations which includes multiplying, dividing and remaindering (Turing machines need more time for this).

EXPECTATIONS

Each student in the group should pick one machine model and compare it to the others which will be discussed in class. Machine models are counter machines (weakest with respect to speed), one-tape Turing machines, multi-tape Turing machines, addition machines on integers, solo-adders (can only set registers to constants and copy registers and add registers and compare registers with <, =, > and work on natural numbers), addition machines augmented by parallel Boolean operations on the bits of a binary representation including shifting (dividing by two with detecting the remainder), same machines with additionally multiplication in unit time, automatic register machines (can apply automatic functions as a primitive operation). Each student chooses one model not taken by the other students and should become an expert on the chosen machine model and have some understanding of the other machines models for comparing the models in the capstone project.

PREREQUISITES (at level 3000 or below, with at most one course at level 3000)

Student should have taken a course on discrete mathematics or equivalent; furthermore, any knowledge on theoretical computer science or automata theory is appreciated but not mandatory.

READING REFERENCES

Some background on complexity classes and some of the machine models is found in the following online reference (which also contains quite some information not related to the project, so no need to read everything):
Models and Theory of Automata and Languages:
https://www.comp.nus.edu.sg/~fstephan/fullautomatatheory-pstopdf.pdf
Specific machine models are dealt with in the following publications which are available on the internet:

Robert W. Floyd and Donald E. Knuth. Addition machines. SIAM Journal on Computing, 19(2):329–340, 1990. http://i.stanford.edu/pub/cstr/reports/cs/tr/89/1268/CS-TR-89-1268.pdf
Sanjay Jain, Xiaodong Jia, Ammar Fathin Sabili, Frank Stephan: Addition machines, automatic functions and open problems of Floyd and Knuth. Journal of Computer and System Sciences 136:135-156 (2023), https://arxiv.org/abs/2111.08969
John Case, Sanjay Jain, Samuel Seah, Frank Stephan: Automatic functions, linear time and learning. Logical Methods Computer Science 9(3) (2013), https://lmcs.episciences.org/734
Ziyuan Gao, Sanjay Jain, Zeyong Li, Ammar Fathin Sabili, Frank Stephan: A Computation Model with Automatic Functions and Relations as Primitive Operations. Theoretical Computer Science 924, 94-116 (2022). https://arxiv.org/abs/2201.06836
Wikipedia on Turing machine and the references in there: https://en.wikipedia.org/wiki/Turing_machine
Wikipedia on Counter machine (sometimes called register machine) and the reference in there: https://en.wikipedia.org/wiki/Counter_machine

POSSIBLE TIMINGS

For the supervisor are meetings on either Tuesday 16:00 hrs or Wednesday 10:00 hrs or Wednesday 12:00 hrs or Wednesday 13:00 hrs or Wednesday 14:00 hrs or Thursday 16:00 hrs possible. The final decision for the time takes student opinions and also room availability into account. These group meetings should be 1-2 hours, once per week.