NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

EXAMINATION FOR
Semester 2 AY 2013–2014

GEM1501 – PROBLEM SOLVING FOR COMPUTING

April/May 2014 Time Allowed 2 Hours

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## INSTRUCTIONS TO CANDIDATES

1. This examination paper consists of TEN (10) questions and comprises ELEVEN (11) printed pages, including this page.
2. Answer **ALL** questions within the space in this booklet.
3. This is a **Closed Book** examination.
4. Please write your Matriculation Number below:

   MATRICULATION NO: ______________________________

5. You may use calculators, provided that they do not contain any program or memory content.
6. Every question carries FIVE (5) marks and there are FIFTY (50) marks in total.

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This portion is for examiner's use only

| Qestion | Marks | Remarks | Qestion | Marks | Remarks |
|---|---|---|---|---|---|
| Q01: | | | Q06: | | |
| Q02: | | | Q07: | | |
| Q03: | | | Q08: | | |
| Q04: | | | Q09: | | |
| Q05: | | | Q10: | | |
| | | | Total: | | |

**Question 1 [5 marks]** **GEM 1501**

Explain what a punch card is and for which early machines (before 1900) it was used.

**Question 2 [5 marks]** **GEM 1501**

The following JavaScript program contains syntax and other programming errors in five of its lines. List them out below and explain what is wrong.

```
function calc(x)      // Line 1
  { array a; integer y,z; // Line 2
    z = calc(x);            // Line 3
    for (y=0;y<55555)          // Line 4
      { z = z+y; a[y] = z;        // Line 5
        switch(z)                    // Line 6
          { case 22: z = z+8; break;    // Line 7
            case 24: z = z+y; break;       // Line 8
            case default: z = 22; break; } }  // Line 9
     if (z < 3000) then { return(a); }           // Line 10
     a[0] = z; return(a); }                         // Line 11
```

**Question 3 [5 marks]** **GEM 1501**

Consider the following JavaScript program.

```
function f(a)
  { var n = a.length;
    var i; var j;
    var b = new Array(n);
    for (i=0;i<n;i++)
      { b[i] = 0;
        for (j=i;j<n;j++)
          { b[i] += a[j]; } }
    return(b); }
```

What is the time complexity of this program?

☐ $\Theta(n)$ ☐ $\Theta(n \cdot \log(n))$ ☐ $\Theta(n^2)$ ☐ $\Theta(n^2 \cdot \log(n))$

This program can be improved significantly. What is the time complexity of your improved program (it should be better than the above)?

☐ $\Theta(\log(n))$ ☐ $\Theta(n)$ ☐ $\Theta(n \cdot \log(n))$ ☐ $\Theta(n^2)$

Write your program below.

**Question 4 [5 marks]** **GEM 1501**

Let $a \oplus b \oplus c$ be 1 if and only if an odd number of the input variables $a, b, c$ have the value 1 (and similarly for other numbers of inputs). Furthermore, let $a \wedge b$ be 1 if and only if both inputs $a$ and $b$ are 1. Now consider

$$F(a, b, c, d) = (a \wedge b) \oplus (a \wedge c) \oplus (a \wedge d) \oplus (b \wedge c) \oplus (b \wedge d) \oplus (c \wedge d).$$

Describe in words when $F(a, b, c, d)$ is 1 and when it is 0. Furthermore, how many of the binary vectors $(a, b, c, d)$ are mapped to 1?

**Question 5 [5 marks]** **GEM 1501**

Describe the following three machines / automata and point out the differences between them:

- a finite automaton;
- a linear bounded Turing machine (also known as linear bounded automaton);
- a Turing machine (without resource bound constraints).

**Question 6 [5 marks]** **GEM 1501**

The Roman general and politician Gaius Julius Caesar used a cryptographic method to encrypt and decrypt texts. Explain the method and also its vulnerabilities: In particular, utilise the knowledge that "L" is a very frequent letter to decrypt the message "DAHHK SKNHZ!"; give the full value of the message; note that spaces and punctuation marks are not encrypted.

**Question 7 [5 marks]** **GEM 1501**

Write a function sort(a) with the following properties:

- The input a is a dynamical array (with some indices used and others not);
- The output b is an empty array at the beginning and should contain the elements of a in sorted order after the end of the function.

For example, if a[5] is 12, a[8] is 11 and a[17] is 22 and no other elements of a exist, then the function should define the array elements b[0] to be 11, b[1] to be 12 and b[2] to be 22.

```
function sort(a)
  { b = new Array();
```

```
    return(b); }
```

**Question 8 [5 marks]** **GEM 1501**

Make a deterministic finite automaton using the alphabet $\Sigma = \{0, 1, 2\}$ such that the automaton accepts a word $w$ if and only if the word contains exactly two times a 1 and exactly one time a 2.

**Question 9 [5 marks]** **GEM 1501**

The following regular expression

$$30^* \mid 20^*10^* \mid 10^*20^* \mid 10^*10^*10^*$$

describes a set of decimal numbers. Give a description of this set in words and list all elements of length one or two. For example, $10^*$ describes the set of all powers of 10 and the elements up to length five are 1 (one), 10 (ten), 100 (hundred), 1000 (one thousand) and 10000 (ten thousand).

**Question 10 [5 marks]** **GEM 1501**

A solution of a set of clauses is a truth-assignment to the logical variables such that each clause is satisfied by the given assignment. The satisfiability problem asks whether for a given set of clauses there is at least one truth-assignment making all clauses true. Consider the following set of clauses:

- $x_1 \vee x_2 \vee x_3 \vee x_5$;
- $\neg x_1 \vee x_4$;
- $\neg x_2 \vee x_4$;
- $\neg x_3 \vee x_4$;
- $\neg x_4 \vee \neg x_5$;
- $x_5 \vee x_6 \vee x_7$;
- $x_5 \vee \neg x_6$;
- $x_5 \vee \neg x_7$.

How many solutions does this set of clauses have?

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 or more.

Write a few lines on how you determined the number of solutions.

END OF PAPER