Homework for 09.09.2004

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Homework. The homework follows the lecture notes. What cannot be done as scheduled, will be done the week afterwards. This homework contains the new tasks for 09.09.2004.

Lecture is Mon 16.00h - 17.30h and Thu 16.00h - 16.45h. Tutorial is Thu 16.45h - 17.30h. The room is S13#05-03.

http://www.comp.nus.edu.sg/~fstephan/homework.ps
http://www.comp.nus.edu.sg/~fstephan/homework.pdf

Exercise 4.8. Which of the following sets is transitive and which is inductive?

1. $A = \{\emptyset, \{\emptyset\}\}$,
2. $B = \{\emptyset, \{\{\{\emptyset\}\}\}\}$,
3. $C = \{x \mid \forall y \in x \, \forall z \in y \, (z = \emptyset)\}$,
4. D is the closure of $\{\emptyset, \mathbb{N} \times \mathbb{N}\}$ under the successor operation $x \mapsto S(x)$,
5. E is the set of even numbers,
6. F is the set of all natural numbers which can be written down with at most 256 decimal digits,
7. G is the set of all finite subsets of $\mathbb{N}$,
8. $H = \mathcal{P}(G)$.

Exercise 4.9. Show that the following statements are equivalent for any inductive set X.

1. $X = \mathbb{N}$;

2. X has no proper inductive subset;

3. X is a subset of every inductive set;

4. $\forall x \in X\,(x = 0 \lor \exists y \in X\,(x = S(y)))$;

5. $X = N(Y)$ for every inductive set Y where $N(Y)$ is the subset of those $y \in Y$ which are in every inductive subset of Y;

6. $X = N(Y)$ for some inductive set Y where $N(Y)$ is defined as in the previous item.

Exercise 4.11. Assume that a property p satisfies

$$p(1) \text{ and } \forall x\,(p(x) \Rightarrow p(S(S(x)))).$$

Consider the following subsets of natural numbers:

1. the set of all numbers;

2. the set of even numbers;

3. the set of odd numbers;

4. the set of square numbers;

5. the set of all powers of 35, that is, $\{1, 35, 1225, 42875, 1500625, \ldots\}$.

For which of these sets it is guaranteed that all elements and for which it is guaranteed that some elements satisfy p? Consider the property

$$q(x) \Leftrightarrow \forall A \subseteq \mathbb{N}\,((1 \in A \land \forall y \in \mathbb{N}\,(y \in A \Rightarrow S(S(y)) \in A)) \Rightarrow x \in A).$$

Which of the above sets is equal to $\{x \in \mathbb{N} \mid q(x)\}$. That is, which set is the intersection of all sets A for which

$$1 \in A \land \forall y \in \mathbb{N}\,(y \in A \Rightarrow S(S(y)) \in A)$$

is true?