Assignment for 09.03.2005. Can be corrected on request, it is not obligatory to hand the homework in.

1. **Diagonal Method.** Prove that there is a set which is neither recursively enumerable nor has a recursively enumerable complement.

2. **S-m-n Theorem.** Prove that there is a recursive function which maps every $e$ to a program computing $e^x$ for input $x$. Give the program explicitly with two inputs $e, x$ and then apply the s-m-n theorem.

3. **Universal Function.** There is a two-place function $\Psi$ such that $\Psi(x, y) = \phi_x(y)$, that is, $\Psi(x, y)$ takes the same value as the $x$-th function on input $y$ and is undefined iff the $x$-th function on input $y$ is undefined. Is the set $\{(x, y) : \psi(x, 5 \cdot y) \downarrow\}$ recursive? Prove your answer.

4. **Closure Properties.** Let $f$ be a total and computable function. Is the set $U = W_{f(0)} \cup W_{f(1)} \cup W_{f(2)} \cup \ldots$ recursively enumerable?