MA 3219 – Computability Theory

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Assignment for 09.03.2005. Can be corrected on request, it is not obligatory to hand the homework in.

1. Diagonal Method. Prove that there is a set which is neither recursively enumerable nor has a recursively enumerable complement.

2. S-m-n Theorem. Prove that there is a recursive function which maps every e to a program computing e^x for input x. Give the program explicitly with two inputs e, x and then apply the s-m-n theorem.

3. Universal Function. There is a two-place function Ψ such that $\Psi(x, y) = \phi_x(y)$, that is, $\Psi(x, y)$ takes the same value as the x-th function on input y and is undefined iff the x-th function on input y is undefined. Is the set $\{(x, y) : \psi(x, 5 \cdot y) \downarrow\}$ recursive? Prove your answer.

4. Closure Properties. Let f be a total and computable function. Is the set $U = W_{f(0)} \cup W_{f(1)} \cup W_{f(2)} \cup \ldots$ recursively enumerable?