MA 3219 – Computability Theory

Frank Stephan. Departments of Computer Science and Mathematics, National University of Singapore, 3 Science Drive 2, Singapore 117543, Republic of Singapore

Email fstephan@comp.nus.edu.sg
Webpage `http://www.comp.nus.edu.sg/~fstephan/computability.html`
Telephone office +65-6874-7354
Office room number S14#06-06
Office hours Thursday 15.00-17.00h

Assignment for 30.03.2005. Can be corrected on request, it is not obligatory to hand the homework in.

1. Recursively Enumerable Sets. Let A be recursively enumerable and undecidable set and B be any set such that the symmetric difference $A \Delta B = (A \cup B) - (A \cap B)$ is finite. Show that B is then also recursively enumerable and that $B \leq_m A$.

2. Productive Sets. Consider the set $E = \{e : W_e \text{ is recursive}\}$. Show that E is productive by constructing a function f such that

$$e \notin K \Leftrightarrow f(e) \in E$$

and applying an appropriate theorem of the lecture. Show furthermore that also the complement $\overline{E} = \{e : W_e \text{ is not recursive}\}$ is productive.

3. Creative Sets. Let π be a computable bijection from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$ and let A and B be creative sets. Consider the following sets C, I and U:

- $C = \{x : x \notin A\}$;
- $I = \{\pi(x, y) : x \in A \text{ and } y \in B\}$;
- $U = \{\pi(x, y) : x \in A \text{ or } y \in B\}$.

Which of these three sets are creative. Prove your answers.

4. Simple Sets. (a) Show that the set $\{x : \exists e\, \exists y\, (\phi_e(y)\downarrow = x \wedge 2 \cdot (e + y)^2 < x)\}$ is simple.

(b) Prove that for every simple set A and every r.e. set B, either $A \cap B$ is infinite or B is finite.

(c) Let A be simple. Prove that there is a set $B \leq_m A$ such that B is neither simple nor recursive.

(d) Let A be simple, $a \notin A$ and $B = A \cup \{a\}$. Prove that $A \not\leq_1 B$, that is, prove that there is no injective, total and computable function f such that, for all x, $A(x) = B(f(x))$. Note that injectiveness of f is essential in this proof since $A \leq_m B$.