Assignment for 06.04.2005. Can be corrected on request, it is not obligatory to hand the homework in.

1. Many-One Reduction. Assume that $A \leq_m C$ and $B \equiv_m C$. Which of these properties are then inherited to $A$ or to $B$ or to both:
   (a) $C$ is recursively enumerable;
   (b) $C$ is simple;
   (c) $C$ is creative;
   (d) $C$ is semirecursive;
   (e) $C$ is recursive.
Here the set $C$ is semirecursive if there is a recursive linear ordering $\sqsubseteq$ such that whenever $x \sqsubseteq y$ and $x \in C$ then also $y \in C$.

2. Turing Reduction. Which two of the following sets are Turing equivalent? Prove this equivalence:
   (a) $\emptyset$;
   (b) $\{e : \phi_e(2345) \downarrow \downarrow = 8\}$;
   (c) $\{e : \phi_e(0) \uparrow \lor \phi_e(1) \uparrow\}$;
   (d) $\{e : \phi_e \text{ is total}\}$;
   (e) $\{e : \phi_e \text{ is infinitely often undefined}\}$.

3. Special Cases. Prove that $\emptyset$, $\mathbb{E}$ and $\mathbb{N}$ Turing equivalent but not many-one equivalent; $\mathbb{E}$ is the set of even natural numbers.

4. Classifying Reductions. Which of the following reductions are many-one reductions and Turing reductions; if a reduction is both then state that it is a many-one reductions:
   (a) $x \in A \iff x \not\in B$;
   (b) $x \in A \iff 2x \in B \land 2x + 1 \notin B$;
   (c) $x \in A \iff 256^x \in B$;
   (d) $x \in A \iff \forall y (2^x \cdot 3^y \in B)$;
   (e) $x \in A \iff y \text{ is even for the least } y \text{ with } 2^x \cdot 3^y \in B$.
At (e) it is assumed that for every $x$ there is a $y$ with $2^x \cdot 3^y \in B$.

5. Growth of functions. Let $A, B$ be r.e. sets and assume that for every total $A$-recursive function $f$ there is a total $B$-recursive function $g$ such that $\forall x \ (f(x) \leq g(x))$. Prove that then $A \leq_T B$. 
