MA 3219 – Computability Theory

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Assignment for 13.04.2005. Can be corrected on request, it is not obligatory to hand the homework in.

1. Examples for recursive operators. Which of the following operators are recursive? Justity your answer.

$$\begin{aligned}
\Psi_1(f)(x) &= \mu y > x(f(y) = 0); \\
\Psi_2(f)(x) &= K(x); \\
\Psi_3(f)(x) &= f(f(f(x))); \\
\Psi_4(f)(x) &= \begin{cases} f(x)+1 & \text{if } f(x) \text{ is defined;} \\ 0 & \text{otherwise;} \end{cases} \\
\Psi_5(f)(x) &= \begin{cases} x^2 + f^2(x) & \text{if } f(x) \text{ is defined;} \\ \uparrow & \text{otherwise.} \end{cases}
\end{aligned}$$

2. Operators and Turing reducibility. Prove the following statement: If $A \leq_T B$ then there is a recursive operator such that $A = \Phi(B)$. Does the converse also hold: If Φ is a recursive operator and $A = \Phi(B)$, is then $A \leq_T B$?

3. First Recursion Theorem. The first recursion theorem says that for every operator Ψ there is a computable function f with $\Psi(f) = f$. Determine fixed points f for the following operators with

$$\begin{aligned}
\Psi_6(f)(x) &= \begin{cases} f(x)+1 & \text{if } f(x) \text{ is defined;} \\ \uparrow & \text{otherwise;} \end{cases} \\
\Psi_7(f)(x) &= f(x+1); \\
\Psi_8(f)(x) &= \begin{cases} f(x-2) + f(x-1) & \text{if } x \geq 2; \\ 1 & \text{if } x < 2. \end{cases}
\end{aligned}$$

4. Construction. Construct an operator where the least fixed-point is the function mapping every even number to 0 and being undefined on odd numbers.